1. A particle of total mass \( m \) is released at a height 1 m, starting from rest and allowed to fall vertically without any friction. At the same time it is released at \( t=0 \), an internal explosion causes two pieces, with masses \( \frac{1}{3} \) kg and \( \frac{2}{3} \) kg to fly off. Immediately after the explosion, the smaller mass traveled at 45° as shown in the diagram with a speed of 1 m/s.

![Diagram of particle explosion](image)

a. (2) Find the velocity of the second mass and sketch it on the figure.

b. (3) Estimate the distance that the smaller mass travels along the horizontal axis when it reaches the ground. Use \( g=10\text{m/s}^2 \) and results within 10% of exact values are acceptable.

---

a. (3) A beam of light with wavelength \( \lambda \) is incident on a surface at an angle \( \phi \) shown in the diagram. It is reflected specularly by two successive planes spaced by \( d \) and gets projected on a screen some distance from the object. Derive the condition for the constructive interference of the wave.

![Diagram of constructive interference](image)
b. (3 points) Suppose that you used blue light of wavelength 480 nm on a lattice and found 4 constructive peaks, in particular by one being at $\phi=90^\circ$ and another one at $\phi=30^\circ$. What is the spacing of the planes and what are the other angles?

3. A particle is in an infinite square well of width L. The state of the particle is given by the wavefunction:

\[
\psi(x) = Ae^{\frac{2\pi x}{L}} \quad 0 \leq x \leq \frac{L}{2}
\]
\[
\psi(x) = 0 \quad \frac{L}{2} < x \leq L
\]

a. (2) What is the value of the constant A?

b. (3 points) Calculate the expectation value of position $\langle \hat{x} \rangle$ and of momentum $\langle \hat{p} \rangle$ for this state.

c. (4 points) What is the probability that the particle is found in the first excited state?
\[ m_1 \mathbf{v}_1 + m_2 \mathbf{v}_2 = 0 \]

a)

\[ \mathbf{v}_2 = -\frac{m_1}{m_2} \mathbf{v}_1 = -\frac{9/3}{2/3} \mathbf{v}_1 = \frac{3}{2} \mathbf{v}_1 \]

\[ \therefore \quad \mathbf{v}_2 = -\frac{3}{2} \left( -\cos 45^\circ \mathbf{i} + \sin 45^\circ \mathbf{j} \right) = \frac{\sqrt{2}}{2} \mathbf{i} - \frac{\sqrt{2}}{2} \mathbf{j} \]

\[ \mathbf{v}_2 = \frac{\sqrt{2}}{2} \mathbf{i} - \frac{\sqrt{2}}{2} \mathbf{j} \]

b) Frictionless, so that \( d = V_x t = \left( -\frac{\sqrt{2}}{2} \right) t \).

But

\[ -\frac{1}{2} g t^2 + V_x^0 t + h = 0 \]

\[ t^2 - 2 \frac{V_x^0}{g} t - \frac{2h}{g} = 0 \]

\[ t^2 - 2 \left( \frac{\sqrt{2}}{2} \right)^2 t - \frac{2}{10} = 0 \]

\[ t^2 - \frac{\sqrt{2}}{10} t - \frac{2}{10} = 0 \]

\[ t = \frac{1}{2} \left( \frac{\sqrt{2}}{10} \right) + \sqrt{\frac{\frac{1}{2}}{100} + 4 \left( 1 \right) \left( \frac{2}{10} \right)} \]

\[ = \frac{\sqrt{2}}{20} + \sqrt{\frac{1}{200} + \frac{2}{10}} \]

\[ = \frac{\sqrt{2}}{20} + \frac{\sqrt{2}}{10} \]

\[ d = \frac{\sqrt{2}}{20} \left( \frac{\sqrt{2}}{10} \right) m = \frac{1}{10} m \]
a) A series of interference patterns, i.e. alternating series of bright and dark areas.

\[ \text{optical path difference} = 2(d \sin \phi) \]

For interference maxima:

\[ 2d \sin \phi = m \lambda, \quad m \in \mathbb{Z}, \]

b) \( \sin \phi = \frac{m \lambda}{2d} \) so there is a range of possible m values,

\[ \frac{m \lambda}{2d} \leq 1 \Rightarrow m \lambda \leq 2d \]

Since \( m = 4 \Rightarrow 4\lambda = 2d \)

\[ d = 2\lambda = 9600 \text{nm} \]

\[ m = 4: \quad 2(2\lambda) \sin \phi = 4\lambda \Rightarrow \phi = 90^\circ \]

\[ m = 3: \quad 2(2\lambda) \sin \phi = 3\lambda \Rightarrow \phi = \sin^{-1}(3/4) \]

\[ m = 2: \quad 2(\lambda) \sin \phi = 2\lambda \Rightarrow \phi = \sin^{-1}(1/2) = 30^\circ \]

\[ m = 1: \quad 2(\lambda) \sin \phi = \lambda \Rightarrow \phi = \sin^{-1}(1/4), \]
\[ a) \quad \int_0^L x^4 \, dx = 1 \Rightarrow \int_0^{L/2} A^2 \, dx + \int_{L/2}^L 0 \, dx = 1 \]

\[ 1 = A^2 \left( \frac{L}{2} \right) = \left[ A = \sqrt{\frac{2}{L}} \right] \]

\[ b) \quad \langle x \rangle \rangle = \int_0^{L/2} x^4 \, dx \]

\[ = \frac{2}{L} \int_0^{L/2} x^2 \, dx \]

\[ = \frac{2}{L} \left. \frac{x^2}{2} \right|_0^{L/2} = \frac{2}{L} \left( \frac{L^2}{8} \right) \]

\[ \langle x \rangle = \frac{L}{4} \]

\[ c) \quad \langle p \rangle \rangle = \int_0^{L/2} x \, dx \]

\[ = \left. \frac{2}{L} \right|_0^{L/2} \]

\[ = \frac{4\pi \hbar}{L^2} \left( \frac{L}{2} \right) = \left[ \frac{2\pi \hbar}{L} \right] \]

9) **Basis Function for Particle in a Box**

\[ y(x) = \sqrt{\frac{2}{L}} \sin \left( \frac{n\pi x}{L} \right) \]
First excited state \( n = 2 \).

\[
\langle -2 | \phi_2 \rangle = \frac{1}{L} \int_0^{L/2} e^{\frac{i2\pi x}{L}} \sin \left( \frac{2\pi x}{L} \right) dx
\]

\[
= \frac{1}{L} \int_0^{\frac{L}{2}} e^{\frac{i2\pi x}{L}} \left( e^{\frac{i2\pi x}{L}} - e^{-\frac{i2\pi x}{L}} \right) dx
\]

\[
= \frac{1}{iL} \int_0^{\frac{L}{2}} e^{\frac{i4\pi x}{L}} dx - \int_0^{\frac{L}{2}} e^{\frac{-i4\pi x}{L}} dx
\]

\[
= \frac{1}{iL} \left[ \int_0^{\frac{L}{2}} e^{\frac{i4\pi x}{L}} dx - \int_0^{\frac{L}{2}} e^{\frac{-i4\pi x}{L}} dx \right]
\]

\[
= -\frac{1}{2i}
\]

\[
\Rightarrow |c|^2 = \rho = \frac{1}{4}
\]