1. (7 points) A point mass, a ring of radius $R$, and a disk of radius $R$ are released from the top of an inclined plane making an angle $\theta$ with the horizontal. All have the same mass, $m$. Assume that the point mass slides down without friction, while the others roll down without slipping.

   a. (2 points) What order will they arrive at the bottom and why?

   b. (5 points) If the speed of the point mass at the bottom is 1 m/s, what is the translational speed of the ring and the disk? Recall that the general expression for the moment of inertia is $I = \int r^2 \, dm$.

2. (3 points) A beam of light is incident upon a glass panel of thickness, $t$, and refractive index $n$, at angle $\theta$, as illustrated.

Assuming that the angle $\theta$ is small, derive a formula for the lateral displacement, $x$, relative to the original path. Express your answer in terms of $\theta$, $t$ and $n$. 

![Diagram of light beam and glass panel](image-url)
3. (4 points) Two immiscible, incompressible fluids are placed inside a U-tube with initial levels shown in the figure with the top openings sealed. The lighter fluid has a density of 3 kg/m$^3$, and the darker fluid has a density of 4 kg/m$^3$. Draw the resulting configuration when the openings are unsealed and calculate the height difference between the fluids. Assume that the pressure of the environment is constant at 1 ATM.

![Diagram of U-tube with two fluids]

4. In a region of space, a particle with zero energy is given by a wavefunction 
   \[ \psi(x) = Cx e^{-x^2/L^2} \], where C is the normalization constant.
   
   a. (4 points) Find the potential energy as a function of x.
   
   b. (2 points) Sketch the potential energy as a function of x, identify the energy level and the classical turning points.
1. (7 points) A point mass, a ring of radius R, and a disk of radius R are released from the top of an inclined plane making an angle $\theta$ with the horizontal. All have the same mass, m. Assume that the point mass slides down without friction, while the others roll down without slipping.

(a) (2 points), what order will they arrive at the bottom and why?

Solution:
$1^{st}$. Point mass, $2^{nd}$. Disk, $3^{rd}$. Ring. The point mass is first because potential energy is entirely converted to translational energy, ring is last because all potential energy is used for rotational motion and disk is $2^{nd}$ because it a mixture, at least some part (center) is purely translational.

(b) (5 points) If the speed of the point mass at the bottom is 1 m/s, what is the translational speed of the ring and the disk? Recall that the general expression for the moment of inertia is $I = \int (r^2) dm$.

Solution:

\[
\frac{1}{2}mv^2 + \frac{1}{2}Iw^2 + mgv = \text{const}
\]

\text{Point mass}

\[
\frac{1}{2}mv^2 = mgh \rightarrow gh = \frac{1}{2}\left(\frac{m}{s}\right)^2
\]

\text{Ring}

\[
\frac{1}{2}mv^2 + \frac{1}{2}Iw^2 = mgh,
\]

\[
I = \int_0^R m (rR)^2 dr = mR^2
\]

\[
\frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{mR^2}{R}\right)^2 = mv^2_R = mgh \rightarrow v_R = \sqrt{gh} = \frac{1}{\sqrt{2}} \frac{m}{s}
\]

\text{Disc}

\[
I = \int_0^R \frac{m}{R^2}r^2 2rdr = \frac{mR^2}{2}
\]

\[
\frac{1}{2}mv_d^2 + \frac{1}{2}\left(\frac{mR^2}{2R}\right)^2 = \frac{3}{4}mv_d^2 = mgh \rightarrow v_d = \sqrt{\frac{4gh}{3}} = \sqrt{\frac{2m}{3}}
\]
2. A beam of light is incident upon a glass panel of thickness, \( t \), and refractive index \( n \), at angle \( \theta \), as illustrated.

\[
n_{\text{air}} = 1 \quad n \quad n_{\text{air}} = 1
\]

\[
\theta \quad t \quad x
\]

(3 points) Assuming that the angle \( \theta \) is small, derive a formula for the lateral displacement, \( x \), relative to the original path. Express your answer in terms of \( \theta \), \( t \) and \( n \).

Solution:

\[
\sin \theta = n \sin \theta' \quad \Rightarrow \quad n' = n
\]

\[
x = \frac{t}{\cos \theta'} \sin \theta' \approx t \left( \frac{1}{n} \right) = t \left( 1 - \frac{1}{n} \right)
\]

b. (4 points) Two immiscible, incompressible fluids are placed inside a U-tube with initial levels shown with the top lids closed. One fluid has a density of 3 kg/m\(^3\) and the other is 4 kg/m\(^3\). Draw the resulting configuration and calculate the height difference.

\[
\text{1 m}
\]
Solution:
Drawing: 1 point, Calculation: 3 points

Pressure must be the same weight at AA'

\[
\frac{m_{\text{black}} g}{A} = \frac{m_{\text{gray}} g}{A} \quad \Rightarrow \quad \frac{\text{black} A (1 - x) g}{A} = \frac{\text{gray} A (1) g}{A}
\]

\[
(1 - x) = \frac{3}{4} \quad \Rightarrow \quad x = \frac{1}{4} m
\]

Solution:
3. In a region of space, a particle with zero energy is given by a wavefunction 
\( y(x) = Cx e^{-x^2/L^2} \), where C is the normalization constant.

a. (4 points) Find the potential energy as a function of x.

b. (2 points) Sketch the potential energy as a function of x, identify the energy level 
and the classical turning points.

Solution:

a. From Schrödinger's Equation:

\[ \frac{\hbar^2}{2m} \frac{d^2}{dx^2} y(x) + V(x) y(x) = E y(x) = 0 \]

\[ \frac{d^2}{dx^2} y(x) = Ax e^{-x^2/L^2} \left( \frac{4x^2}{L^4} - \frac{6}{L^2} \right) = V(x) y(x) \]

\[ V(x) = \frac{\hbar^2}{2m} \left( \frac{4x^2}{L^4} - \frac{6}{L^2} \right) \]

b.