1. A parallel-plate capacitor of width w, length L, and separation d is partially filled with a dielectric medium of relative dielectric constant $\varepsilon_r$, as shown in Fig. below. A battery of $V_0$ volts is connected between the plates. Region 1 is the region between the plates where the relative dielectric constant is $\varepsilon_r$, region 2 is the region between the plates where the dielectric constant is $\varepsilon_r=1$.
   
   a. (3 pts) Find $\mathbf{D}$, $\mathbf{E}$, and $\rho_s$ in region 1 and region 2.
   
   b. (3 pts) Find distance $x$ such that the electrostatic energy stored in each region is the same.

![Diagram of a capacitor with dielectric](image)

2. Consider a conducting bar (see Fig. below) sliding at velocity $u_z$ over parallel rails oriented along the $z$-direction in a time-varying magnetic field generated by an infinitely long wire carrying a current oscillating as:

$$i = i_0 \cos(\omega t)$$

![Diagram of a conducting bar](image)

a. (2 pts) Give an expression for the magnetic field a distance $r$ away from the wire located at $x = 0$ and carrying the current $i$.

b. (2 pts) Give an expression for the $V_{emf}$ induced between points a and b.

c. (2 pts) Take the current as flowing upward at time $t = 0$. In which direction is the current flowing in the rails at time $t = 0$ if a resistance $R$ is introduced between points a and b. Explain your answer.
3. Consider a plane wave incident at normal incidence to an interface between vacuum and a dielectric stack of two non-magnetic materials of index $n_2$ and $n_3$. The thickness of the first film of index $n_2$ is $d$.

a. (2 pts) Using the appropriate boundary conditions, derive the amplitude reflection coefficient $r_{12}$ for a plane wave representing the electric field incident from medium 1 onto medium 2:

$$r_{12} = \frac{n_1 - n_2}{n_1 + n_2}.$$

b. (2 pts) Write an expression for the phase difference between a wave reflecting at the interface between medium 1 and 2 and a plane wave going through at the interface of media 1 and 2 but being reflected at the interface of media 2 and 3 and coming back out through the interface between media 1 and 2.

c. (1 pt) What is the condition on the thickness $d$ for having destructive interference between the two waves of part (b).

d. (2 pt) What is the condition on the indices of refraction to ensure zero reflectivity of the incident wave from media 1 and 2 combined?
1. Region 1 is the dielectric and region 2 is the air.

\( \overrightarrow{E}_1 = -a_1 \frac{V_0}{d}, \quad \overrightarrow{D}_1 = -a_1 \varepsilon_0 \varepsilon_r \frac{V_0}{d}, \quad \rho_{s1} = \varepsilon_0 \varepsilon_r \frac{V_0}{d} \) (top plate)

a. \( \overrightarrow{E}_2 = -a_2 \frac{V_0}{d}, \quad \overrightarrow{D}_2 = -a_2 \varepsilon_0 \varepsilon_r \frac{V_0}{d}, \quad \rho_{s2} = \varepsilon_0 \frac{V_0}{d} \) (top plate)

b. \( \frac{W_{c1}}{W_{c2}} = \frac{\varepsilon_r x}{L - x} = 1 \Rightarrow x = \frac{L}{\varepsilon_r + 1} \)

2. a. Using Ampere’s law, we get:

\[ B = \frac{\mu_0 I_0 \cos(\omega t)}{2\pi r} ; \quad \overrightarrow{H} = \frac{B}{\mu_0} \overrightarrow{y} \]

b. \[
V = -\frac{d}{dt} \left( \int \overrightarrow{B} \cdot d \overrightarrow{s} \right) = -\frac{d}{dt} \left( \int \mu_0 I_0 \frac{\cos(\omega t)}{2\pi} \right) r_z i \mu_0 I_0 \frac{\cos(\omega t)}{2\pi} \frac{dr}{r} \\
V = -\frac{\mu_0 I_0}{2\pi} \ln \frac{r_2}{r_1} u_z \frac{d}{dt} \left( t \cos(\omega t) \right) \\
V = \frac{\mu_0 I_0}{2\pi} \ln \frac{r_2}{r_1} u_z \left( -\cos(\omega t) + t \omega \sin(\omega t) \right) \]

c. At \( t = 0 \), we have

\[ V_{emf} = -\frac{\mu_0 I_0}{2\pi} \ln \frac{r_2}{r_1} u_z = V_b - V_a, \]

a negative quantity. The current in the rails flows in a counterclockwise direction. The generated current creates a magnetic flux between the rails which is increasing so to compensate for the decreasing flux.

3. a. At \( z = 0 \), the tangential components of \( \overrightarrow{E} \) and \( \overrightarrow{B} \) are continuous.

\[ E_i + E_r = E_i \]

and

\[ \overrightarrow{B} = -\frac{\nabla \times \overrightarrow{E}}{\omega} = iE_k \frac{k_i}{\omega} \overrightarrow{y} \]

\[ E_k \frac{k_i}{\omega} = E_r \frac{k_r}{\omega} = E_i \frac{k_i}{\omega} = (E_i + E_r) \frac{k_i}{\omega} \]

But \( k_r = k_i \) and \( \frac{k_i}{k_j} = n \). Taking \( n_1 = 1 \),
\[ E_r (1+n) = E_i (1-n) \]
\[ r = \frac{E_r}{E_i} = \frac{1-n}{1+n} \]

b. \[ \Delta \phi = \frac{4\pi n_2 d}{\lambda_0} \], where \( \lambda_0 \) is the wavelength of light in vacuum.

c. We have destructive interference when \( \Delta \phi = (2m+1)\pi \), where \( m \) is an integer. In particular, for \( m = 0 \), we have destructive interference when \( d = \frac{(2m+1)\lambda}{4} = \frac{\lambda}{4} \), where \( \lambda \) is the wavelength of light in the medium 2.

d. In order to get both reflections to be of the same size, we need to have
\[
\frac{n_1 - n_2}{n_1 + n_2} = \frac{n_2 - n_3}{n_2 + n_3} \quad \text{or for } n_1 = 1,
\]
\[ n_2 = \sqrt{n_3} \]