Part (i) (7 pts.)
Consider a system whose input $x(t)$ has a continuous derivative, and whose output $y(t)$ equals that derivative:

$$y(t) = \frac{d}{dt} x(t)$$

Is this system:
(a) (1 pt.) Linear?
(b) (2 pts.) Time-invariant?
(c) (2 pts.) Causal?
(d) (2 pts.) Bounded-input bounded-output (BIBO) stable?
Justify your answers.

Part (ii) (7 pts.)
Consider a continuous-time linear time-invariant (LTI) system. When the input to this system is

$$x(t) = e^{-2t+3}u(t-4)$$

the output is $y(t)$; and when the input is

$$\frac{d}{dt} x(t)$$

the output is

$$-2y(t) + e^{-t}u(t)$$

Find the impulse response, $h(t)$, of this LTI system.

Part (iii) (6 pts.)
A real-valued signal sequence $x[n]$ is such that $x[1] > 0$. Also, its discrete-time Fourier transform is given by

$$X(e^{j\omega}) = \begin{cases} 
    c, & \omega \in (0, \pi); \\
    0, & \omega = 0, \pi; \\
    -c, & \omega \in (-\pi, 0).
\end{cases}$$

where $c$ is a constant such that $|c| = 1$. Is $c$ purely real, purely imaginary, or does it have both real and imaginary parts? Determine $x[n]$ for every $n$. 

Part (i)
This system is:

(a) Linear, because
\[ \frac{d}{dt} (a_1 x_1(t) + a_2 x_2(t)) = a_1 \frac{d}{dt} x_1(t) + a_2 \frac{d}{dt} x_2(t) \]

(b) Time-invariant, because
\[ \frac{d}{dt} x(t - \tau) = \left( \frac{d}{dt} x \right) (t - \tau) \]

(c) Causal, because, \( x(t) \) is continuously differentiable, and for \( \delta \geq 0 \),
\[ y(t) = \frac{d}{dt} x(t) = \lim_{\delta \to 0} \frac{f(t + \delta) - f(t)}{\delta} = \lim_{\delta \to 0} \frac{f(t) - f(t - \delta)}{\delta} \]
and the output at time \( t \) depends only on the input at time \( \tau \leq t \).

(d) NOT bounded-input bounded-output (BIBO) stable, because, for instance, the bounded input \( x(t) = \sin(t^2) \) yields an unbounded output \( y(t) = 2t \cos(t^2) \).

Part (ii)
Note that
\[ \frac{d}{dt} x(t) = -2e^{-2t+3}u(t - 4) + e^{-5}\delta(t - 4) = -2x(t) + e^{-5}\delta(t - 4) \]
and the corresponding output is stated to be
\[ -2y(t) + e^{-t}u(t) \]

Therefore, the input
\[ e^{-5}\delta(t - 4) \]
yields the output
\[ e^{-t}u(t) \]

From here, using the properties of LTI systems, we conclude that, the input \( \delta(t) \) yields the output
\[ e^{-(t-1)}u(t+4) \]
which is the impulse response of this LTI system.
Part (iii)
For real valued signals,
\[ X(e^{jw})^* = X(e^{-jw}) \]
and therefore, for this problem, \( c^* = -c \), i.e., \( c \) is purely imaginary. Using \( |c|=1 \), \( c \) is either \( +j \) or \( -j \). Then,

\[
x[n] = \frac{1}{2\pi} \int_{-\pi}^{\pi} X(e^{jwn})e^{jwn} \, dw = \frac{1}{2\pi} \int_{-\pi}^{0} (-c)e^{jwn} \, dw + \frac{1}{2\pi} \int_{0}^{\pi} ce^{jwn} \, dw
\]

\[
= \frac{c \frac{1}{\pi n}}{j} ((-1)^n - 1)
\]

Using \( x[1] > 0 \), we conclude \( c = -j \). Then, \( x[n] \) becomes

\[
x[n] = \frac{1}{\pi n} (1 - (-1)^n)
\]