Part (i) (7 pts.)
Let \( u(t) \) be the unit step function in continuous time. When the input
\[
x(t) = e^t u(-t)
\]
is applied to a linear time-invariant (LTI) system, the resulting output is given by
\[
y(t) = e^{-t}u(t) - 2e^{-3t}u(t)
\]
Determine the impulse response \( h(t) \) of the system.

Part (ii) (7 pts.)
The continuous-time signal \( f(t) \) has a real-valued Fourier transform \( F(j\omega) \), as shown in the figure below. \( F(j\omega) \) is nonzero for \( 300 < \omega/(2\pi) < 900 \) only.

\[
\begin{array}{c}
\text{\( F(j\omega) \)} \\
\hline
0 & 300 & 600 & 900 & \omega/(2\pi) \\
\end{array}
\]

a. (2 pts.) Which, if any, of the following identities are true for all \( t \in \mathbb{R} \)? Explain briefly.
\[
\begin{align*}
f(t) &= f^*(t) \\
f(t) &= f(-t) \\
f(t) &= f^*(-t)
\end{align*}
\]
b. (1 pt.) Does there exist an interval \( I \) of finite length such that \( f(t) = 0 \) for \( t \notin I \)? Explain briefly.
c. (4 pts.) If
\[
g(t) = f(-t/3) \sin(400\pi t)
\]
sketch the real and imaginary parts of the Fourier transform \( G(j\omega) \) against \( \omega/(2\pi) \), labeling each graph fully.

Part (iii) (6 pts.)
The impulse response of a discrete-time LTI system is given by
\[
h[n] = \sum_{k=0}^{\infty} \alpha^k \delta[n - 3k],
\]
where \( \alpha \in (0,1) \) and \( \delta[\cdot] \) denotes the unit impulse function.

a. (1 pt.) Sketch \( h[n] \).

b. (2 pts.) Determine coefficients \( \{a_k\} \) and \( \{b_k\} \) such that the system can be realized using the difference equation
\[
\sum_{k=0}^{N} a_k y[n - k] = \sum_{k=0}^{M} b_k x[n - k]
\]
c. (3 pts.) Two identical systems, each with impulse response \( h[n] \), are connected in cascade (series). Determine the impulse response \( g[n] \) of the resulting cascade.
(i) Using Laplace transforms:

\[ X(s) = -\frac{1}{s-1} \quad ; \quad \text{ROC: } \text{Re}\{s\} < 1 \]

\[ Y(s) = \frac{1}{s+1} - \frac{2}{s+3} \]

\[ = -\frac{(s-1)}{[(s+1)(s+3)]} \quad ; \quad \text{ROC: } \text{Re}\{s\} > -1 \]

Therefore

\[ H(s) = \frac{Y(s)}{X(s)} \]

\[ = \frac{(s-1)^2}{[(s+1)(s+3)]} \]

\[ = 1 + \frac{2}{s+1} - \frac{8}{s+3} \quad ; \quad \text{ROC: } \text{Re}\{s\} > -1 \]

\[ h(t) = \delta(t) + (2\exp(-t) - 8\exp(-3t))u(t) \]

(ii)

(a) \( F(jw) \) is real-valued and has no symmetry about the origin \( w=0 \). It follows that \( f(t) \) is complex-valued, with conjugate even symmetry about \( w=0 \):

\[ f(t) = f(-t) \]

(Only third identity is true.)

(b) Since \( F(jw) \) is bandlimited, \( f(t) \) cannot be time-limited (i.e., vanish outside a finite time interval). So no such finite interval \( I \) exists.

(c)

\[ f(t) \quad \longleftrightarrow F(jw) \]

\[ f(-t/3) \quad \longleftrightarrow 3F(-j*3*w) \]

\[ f(-t/3)*\sin(400*\pi*t) \quad \longleftrightarrow G(jw) \]

where

\[ (1/3)G(jw) = \frac{j}{2}F(-j*3*(w+400*\pi)) \]

\[-(j/2)*F(-j*3*(w-400*\pi)) \]

\[ \text{Re}\{G(jw)\} = 0. \quad \text{Im}\{G(jw)\} \text{ consists of two segments over } [-500,-300] \text{ and } [-100,100] \text{ Hz.} \]

(iii)

(a = alpha)

(b) \( y[n]-a*y[n-3] = x[n] \)

(c) \( g[n] = 0 \) for \( n<0 \) or \( n \) not multiple of \( 3*k \)
For $n = 3k \geq 0$,

$$g[n] = \sum (a^r) * (a^{(k-r)} \text{ for } r = 0, \ldots, k)$$

$$= (k+1)a^k$$