Digital Logic – ECE Written Qualifying Examination – Spring 2014

1. (3 points)
   Use the 2’s complement number representation to carry out the following arithmetic operations (the numbers given are decimal). Show the steps and specify the results in decimal.
   (1) 76 + 12
   (2) 75 – 203

2. (3 points)
   A gate or a set of gates is called universal if it can implement all digital system (all Boolean functions). A standard way to show that a gate or a set of gates is universal is to show that it can implement the standard universal set of gates: \{\text{AND, OR, NOT}\}

   Determine whether the gate represented by the following Boolean function is universal. If yes, give the proof. If not, explain why.
   \[
   f(x, y, z) = x'y'z + x'yz' + x'yz + xy'z
   \]

3. (7 points)
   Explain what is a minterm, what is a prime implicant, and what is an essential prime implicant. You can use a 4-variable Boolean function as an example.

   Can you find a 4-variable Boolean function that has more prime implicants than minterms? Can you find a 4-variable Boolean function that has more essential prime implicants than minterms? If yes, give an example. If not, explain why.

4. (7 points)
   Design a sequential circuit with the minimal number of T flip-flops to generate the periodic repetition of binary sequence 0, 1, 3, 5, 7, one digit at each clock cycle. You can assume that the binary sequence is stored in the T flip-flops.
   (1) Draw the state diagram for this system.
   (2) Draw the state transition table for the same system.
   (3) Derive the flip-flop input function for each of the flip-flops in the sum of products format with the minimum number of literals.
   (4) Draw the logic diagram of this sequential circuit.
   (5) When the system is powered on, the initial contents of the flip-flops represent binary value 6. Show the binary sequence generated in the next 20 clock cycles.
1. (3 points)
Use the 2’s complement number representation to carry out the following arithmetic operations (the numbers given are decimal). Show the steps and specify the results in decimal.

(1) $76 + 12$
(2) $75 - 203$

Answer:

$76_{10} = 01001100_2$  
$12_{10} = 00001100_2$

$(76 + 12)_{10} = 01011000_2 = 88_{10}$

$203_{10} = 011001011_2$
$75_{10} = 001001011_2$

$-203_{10} = 100110101_2$

$(75 - 203)_{10} = 001001011_2 + 100110101_2 = 110000000_2 \rightarrow -128_{10}$

2. (3 points)
A gate or a set of gates is called universal if it can implement all digital system (all Boolean functions). A standard way to show that a gate or a set of gates is universal is to show that it can implement the standard universal set of gates: \{AND, OR, NOT\}

Determine whether the gate represented by the following Boolean function is universal. If yes, give the proof. If not, explain why.

\[ f(x,y,z) = x'y'z + x'yz' + x'yz + xy'z \]

Answer:

\[ f(x,y,z) = x'y + y'z \]

\[ \text{NOT: } f(x,1,z) = x' \]
\[ \text{OR: } f(0,y,z) = y + y'z = y + z \]
\[ \text{AND: } f(f(x,1,z), y, 0) = f(x',y,0) = xy \]

So the given gate is universal.

3. (7 points)
Explain what is a minterm, what is a prime implicant, and what is an essential prime implicant. You can use a 4-variable Boolean function as an example.

Can you find a 4-variable Boolean function that has more prime implicants than minterms? Can you find a 4-variable Boolean function that has more essential prime implicants than minterms? If yes, give an example. If not, explain why.

Answer:

A minterm is a product term where each variable appears exactly once, either in true or complemented form.
A prime implicant is a product term with fewer than all the variables in the function. It is only when the function should be 1. If any variable is removed from the term, it will be 1 when the function is not 1.

An essential prime implicant is a prime implicant that is the only prime implicant that can cover one particular minterm of the function.

Consider the following function,
\[ f(w, x, y, z) = w'x'y' + w'x'z + w'yz + xyz + wxz + wxy' + xy'z' + w'yz' + wy'z + x'y'z \]

It has 10 prime implicants, but only 9 minterms.

A function cannot have more essential prime implicants than minterms because each essential prime implicant must have at least one minterm that cannot be covered by other prime implicants.

4. (7 points)

Design a sequential circuit with the minimal number of T flip-flops to generate the periodic repetition of binary sequence 0, 1, 3, 5, 7, one digit at each clock cycle. You can assume that the binary sequence is stored in the T flip-flops.

(1) Draw the state diagram for this system.
(2) Draw the state transition table for the same system.
(3) Derive the flip-flop input function for each of the flip-flops in the sum of products format with the minimum number of literals.
(4) Draw the logic diagram of this sequential circuit.
(5) When the system is powered on, the initial contents of the flip-flops represent binary value 6. Show the binary sequence generated in the next 20 clock cycles.

Answer:

(1) State diagram
(2) State transition table
(3) Truth tables:

\[
\begin{array}{c|cccc}
A & B & C & T_A & T_B \\
\hline
0 & 0 & 0 & 0 & 0 \\
1 & 0 & 1 & 1 & 1 \\
3 & 1 & 1 & 1 & 1 \\
5 & 1 & 1 & 1 & 0 \\
7 & 1 & 1 & 0 & 0 \\
2 & 0 & 1 & 0 & - \\
4 & 1 & 0 & - & - \\
6 & 1 & 1 & - & - \\
\end{array}
\]

\[
\begin{array}{c|cccc}
A & B & C & T_A & T_B \\
\hline
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 1 \\
0 & 1 & 1 & 1 & 1 \\
1 & 0 & 1 & 0 & 1 \\
1 & 1 & 1 & 1 & 1 \\
0 & 1 & 0 & - & - \\
1 & 0 & 0 & - & - \\
1 & 1 & 0 & - & - \\
\end{array}
\]

\[
T_A = B \\
T_B = C \\
T_C = C + AB
\]

(4) Logic diagram

(5) Sequence: 6 → 3 → 5 → 7 → 0 → 1 → 3 → 5 → 7 → 0 → 1 → 3 → 5 → 7 → 0 → 1