1. Consider a charge +q uniformly distributed throughout the volume of a sphere of radius a and placed at the center of a spherical conducting shell of inner radius b and outer radius c. The outer shell carries a net charge –q. Find an expression for the electric field (magnitude and direction):

   a. (2 pts) Within the sphere (r<a),
   b. (2 pts) Between the sphere and the shell (a<r<b),
   c. (1 pt) Inside the shell (b<r<c), and
   d. (1 pt) Outside the shell (r>c)
   e. (1 pt) What charges appear on the inner and outer surfaces of the shell?

2. Let B be a magnetic field pointing into the paper and increasing at the rate dB/dt. Let R be the effective radius of the cylindrical region (the cylinder axis is along the z-direction) in which the magnetic field is assumed to exist (see figure below). Note that \( \vec{a}_z \) is perpendicular and coming out of the page.

   a. (3 pts) What is the magnitude of the electric field \( \vec{E} \) at any radius r?
   b. (2 pt) What is the direction of \( \vec{E} \) and explain why.
   c. (2 pt) Take dB/dt = 0.10 T/s and R = 10 cm. Make a plot of the electric field as a function of r for r up to 40 cm.
3. Assume that the attenuation on a 50 Ω distortionless (characteristic impedance does not depend on frequency) transmission line is 0.01 dB/m (take 1 Neper/m as equal to 8.69 dB/m) and assume that the line has a capacitance of 0.1 nF/m. Show the formulas that you are using and evaluate the results.
   a. (3 pts) Find the resistance, inductance, and conductance per meter of the line.
   b. (2 pts) Find the velocity of wave propagation,
   c. (1 pt) Determine the percentage by which the amplitude of a voltage traveling wave is decreased in 1 km.
1) a) Using Gauss's law:
\[ \oint E \cdot d\mathbf{a} = \frac{Q}{\varepsilon_0} \]

\[ E \times 4\pi R^2 = \frac{q}{\varepsilon_0} \frac{4\pi R^3}{\frac{4}{3} \pi a^3} \]

\[ \Rightarrow \mathbf{E} = \left( \frac{q}{4\pi \varepsilon_0 a^3} \right) \mathbf{\hat{a}}_n \]

b) Similarly, using Gauss's law, neglect:
\[ E^2 = \frac{q}{4\pi \varepsilon_0 R^2} \mathbf{\hat{a}}_n \]

C) \( \mathbf{E} = 0 \) since the charge is zero

D) \( \mathbf{E} = 0 \) since the charge is zero

E) inner \( \rightarrow -q \)
\[ \text{outer} \rightarrow 0 \]
a) For $r < R$, the flux $\Phi$ through a loop of radius $r$ is:

\[ \Phi = B r^2 \]

Using Faraday's law:

\[ \oint E \cdot dl = -\frac{d\Phi}{dt} \]

\[ E \cdot 2\pi r = -\frac{d\Phi}{dt} = (-r^2) \frac{dB}{dt} \]

\[ E = -\frac{1}{2} r \frac{dB}{dt} \rightarrow \bar{E} = \left[ \frac{1}{2} r \frac{dB}{dt} \right] \hat{a}_\phi \]

Where the minus sign indicates that the induced electric field opposes the change of the magnetic flux.

For $r > R$,

\[ \Phi = \int \bar{B} \cdot d\vec{s} = B \pi R^2 \]

From Faraday's law:

\[ \oint E \cdot dl = -\frac{d\Phi}{dt} \]

\[ E \cdot 2\pi r = -\frac{d\Phi}{dt} = -\pi r^2 \frac{dB}{dt} \]

\[ E = -\frac{1}{2} \frac{R^2}{r} \frac{dB}{dt} \rightarrow \bar{E} = \left[ \frac{1}{2} \frac{R^2}{r} \frac{dB}{dt} \right] \hat{a}_\phi \]

(b) When $\bar{B}$ is increasing going into the page, the electric field direction is as shown in Figure.
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a) For a distanceless line:

\[ \frac{R}{L} = \frac{G}{C} \quad \text{where } G \text{ is conductance} \]

\[ R_0 = \sqrt{\frac{L}{C}} = 50 \, \Omega \]

\[ \alpha = R \sqrt{\frac{C}{L}} = 0.01 \, \text{cm} = 1.15 \times 10^{-3} \, \text{m} \]

\[ \Rightarrow R = \alpha R_0 = 0.057 \, \Omega/m \]

\[ L = C R_0^2 = 10^{-10} \times 50^2 = 0.25 \, \mu H/m \]

\[ G = \frac{R C}{L} = \frac{R}{R_0^2} = \frac{0.057}{50^2} = 22.8 \, \mu S/m \]

b) \[ \nu_p = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{0.25 \times 10^{-6} \times 10^{-10}}} = 2 \times 10^9 \, \text{m/s} \]

c) \[ \frac{V_2}{V_1} = e^{-2\alpha} = e^{-1.15} = 0.317 \]