Part (a) (7 pts.)

(a1) (3 pts.) Consider the continuous-time linear time-invariant system with impulse response \( h(t) = e^{-t}u(t) \), where \( u(t) \) is the unit step function. Determine and sketch the output \( y(t) \) of the system when the input is

\[
x(t) = \begin{cases} 
  1, & 0 < t \leq 1 \\
  0, & \text{otherwise}
\end{cases}
\]

(a2) (4 pts.) Determine and sketch the output \( w(t) \) of the same system when the input \( v(t) \) is periodic with period 2 (i.e., \( v(t) = v(t - 2) \) for all \( t \)) and such that

\[
v(t) = x(t), \quad 0 \leq t < 2
\]

with \( x(t) \) as in part (a1). Your expression for \( w(t) \) should not contain infinite sums.

Part (b) (7 pts.)

(b1) (3 pts.) Determine and sketch (real and imaginary parts separately) the Fourier transform \( F(j\omega) \) of the triangular pulse

\[
f(t) = \begin{cases} 
  1 - (|t|/T), & 0 < |t| \leq T; \\
  0, & \text{otherwise}
\end{cases}
\]

(b2) (4 pts.) Consider a continuous-time LTI system with impulse response

\[
g(t) = A \left( \frac{\sin \omega_0 t}{t} \right)^2, \quad t \in \mathbb{R}
\]

Does there exist an input signal \( r(t) \) for which the output of the system is given by

\[
s(t) = e^{-|t|}, \quad t \in \mathbb{R}
\]

If so, what is that input?

Part (c) (6 pts.)

A discrete-time linear time-invariant system has impulse response

\[
h[n] = a\delta[n] + b\delta[n - 1] + \delta[n - 2] + b\delta[n - 3] + a\delta[n - 4],
\]

where \( a \) and \( b \) are both real-valued and, as usual, \( \delta[n] \) is the unit impulse at \( n = 0 \). If the input

\[
x[n] = (-1)^n + \cos(2\pi n/3) \quad (n \in \mathbb{Z})
\]

produces an output which is zero for all \( n \), determine \( a \) and \( b \).
Part (a)

(a1) Clearly, the system is causal. By convolving \( h(t) \) and \( x(t) = u(t)-u(t-1) \), we obtain

\[
y(t) = \begin{cases} 
0, & t < 0 \\
1 - \exp(-t), & 0 \leq t < 1 \\
(e-1)\exp(-t), & t \geq 1 
\end{cases}
\]

\( v(t) \) is the sum of \( x(t-2*k) \) over all \( k \); thus \( w(t) \) is also the sum of \( y(t-2*k) \) over all \( k \). And since \( v(t) \) is periodic with period 2, so is \( w(t) \). For \( 0 \leq t < 2 \),

\[
v(t) = y(t) + \sum \left\{ (1-\exp(-1))\exp(-(t-2*k)) \right\}, k>0
\]

\[
= y(t) + \exp(-t)/(1+e)
\]

More extensively,

\[
v(t) = \begin{cases} 
1 - \exp(-t+1)/(1+e), & 0 \leq t < 1 \\
\exp(-t+2)/(1+e), & 1 \leq t < 2 
\end{cases}
\]

Part (b)

(b1) One can integrate \( f(t)\exp(-j*w*t) \) over \( t \) directly. Alternatively, use the fact that \( T*f(t) \) is the convolution of two identical rectangular pulses of width \( T \) and unit height centered at the origin. Since convolution is equivalent to multiplication in the frequency domain, the result is

\[
F(j*w) = (4/T)(\sin(w*T/2)/w)^2
\]

Since \( f(t) \) is real and even in \( t \), so is \( F(j*w) \) in \( w \).

(b2) By duality (to the result of (b1)), the frequency response \( G(j*w) \) is also real, triangular and symmetric in \( w \), and vanishes for \( |w| > 2*w_0 \). Since the output signal has

\[
S(j*w) = 2/(1+w^2)
\]

for all \( w \), there is no \( r(t) \) such that

\[
S(j*w) = G(j*w)*R(j*w)
\]

Part (c)
The frequency response of the system is given by \((z=\exp(jw))\)

\[
H(z) = z^{-2}(a*z^2 + b*z + 1 + b*z^{-1} + a*z^{-2})
\]

Setting \(H(z) = 0\) for

\(z = -1\) and \(z = \exp(j*2*pi/3)\)

results in

\[
2*a - 2*b + 1 = 0
\]

and

\[
2*a*cos(4*pi/3) + 2*b*cos(2*pi/3) + 1 = -a - b + 1 = 0
\]

Thus \(a = 1/4, b = 3/4\).

Alternatively, one can use the input-output relationship (\(y[n]\) in terms of \(x[n-k]\)'s) for two values of \(n\) to obtain (distinct) linear equations in \(a\) and \(b\).