1. Shown in the figure is a top view of a mass $m$ connected between two springs with spring constants $k_1$ and $k_2$. The mass rests on a surface and is subject to a vertical gravitational force and a horizontal friction force.

A) First assume that friction is negligible and find an expression for the frequency of oscillation. (3 pts)

B) Now suppose that the surface is characterized by a coefficient of kinetic friction $\mu$, which is sufficiently small that the frequency of oscillation is unchanged. How much energy is extracted from the oscillating mass in one cycle of oscillation ($0 < t < 2\pi / \omega$) assuming the excursion is given by $x(t) = x_m \cos(\omega t)$ with $x_m$ approximately constant. (2 pts).

C) Obtain an equation for the decay of the amplitude of oscillation as a function of time due to friction. Give your equation in terms of the frequency of oscillation, the gravitational acceleration constant $g$, and the coefficient of kinetic friction $\mu$(2 pts).
2. The internal energy of an ideal gas is in the form $U = (m / 2)NkT$, where $m$ is the number of “bins” per molecule where thermal energy can be stored, $N$ is the number of molecules, $k$ is Boltzmann’s constant and $T$ is the temperature. The pressure of the gas is given by a similar formula $PV = NkT$.

A. For a gas of monatomic molecules, $m=3$ corresponding to the three components of velocity contributing to the molecule’s kinetic energy. Give an argument for what $m$ might be for a gas of diatomic molecules. That is, give a value for $m$ and explain why that is the case. (2 pts)

B. In a sound wave the gas is compressed adiabatically, that is no heat is transferred from regions of compression to regions of rarefaction. Assume $m$ is given, derive the relation between density and pressure for this process, and show how it depends on $m$. (3 pts)

C. Now suppose that the thermal conductivity of the gas is so large that heat is transferred readily from the region of compression to the region rarefaction. What is the relation between pressure and density in this case? (1 pt)
3. A particle of mass $m$ has quantum wavefunction

$$\psi(x) = A \sin^2(kx) = A(1 - \cos(2kx))/2 \text{ for } 0 < x < \pi/k \text{ (and } \psi = 0 \text{ otherwise).}$$

A. Give an expression for the constant $A$. (2 pts)

B. If the wavefunction is in a stationary state with energy $E$ what is the confining potential? (3 pts)

C. Make a sketch (no math solution needed) of what you expect the wavefunction in the first excited state to look like. (2 pts)
Basic Physics Fall 2014 Qualifying Exam Solutions

1A) \[ m \ddot{x} = F_x = -(k_1 + k_2)x \]

\[ x = x_m \cos(\omega t) \]

\[ \omega^2 = \frac{(k_1 + k_2)}{m} \quad \omega = \sqrt{\frac{k_1 + k_2}{m}} \]

1B) \[ W = \int_{x_1}^{x_2} dx \cdot F_x \quad |F_x| = \mu mg \]

During one cycle \[ \Delta x = 4x_m \]

\[ W = 4x_m \mu mg \]

1c) \[ U = \text{Energy of oscillator} = \frac{1}{2} m \omega^2 x_m^2 \]

\[ \frac{dU}{dt} \sim -\frac{W}{T} = -\frac{\omega}{2\pi} W \]

\[ \frac{dU}{dt} = m\omega^2 x_m \dot{x}_m = -\frac{\omega}{2\pi} 4x_m \mu mg \]

\[ \dot{x}_m = -\frac{2}{\pi} \frac{\mu g}{\omega} \]

\[ x_m(t) = x_m(0) - \frac{2\mu g t}{\pi \omega} \]
2. Two answers are possible depending on whether

A vibrational degree is accessible. If the temperature is low and vibrational modes are
frozen out, \( m = 3 + 2 = 5\), 3 translational degrees
and 2 rotational. If the temperature is high enough
that vibrational levels are populated \( m = 3 + 2 + 2 = 7\),
where the extra 2 are air degrees are associated
with the "free" of vibration.

B \[ \Delta U = Q - PV \text{ first law of Thermodynamics} \]
\[ \Delta U = \frac{m}{2} Nk\Delta T \quad \frac{\Delta U}{Nk\Delta T} \]
\[ PV + V\alpha P = Nk\Delta T \]

So \[ \frac{\alpha m^2}{2} PV = \frac{m+2}{2} P\Delta V \]
\[ \frac{\alpha m^2}{2} \Delta V = -\frac{m+2}{2} P \Delta V \]

or \[ \frac{\Delta P}{P} = \frac{\alpha m^2}{m} \left(\frac{-\Delta V}{V}\right) = -\gamma \frac{\Delta V}{V} \]
\[ \gamma = \frac{m+2}{2} \]

So \[ PV^\gamma = \text{const} \]
\[ \frac{P}{n^\gamma} = \text{const} \quad n = \frac{V}{V} \]

C. \( T = \text{const} \quad \gamma = \frac{5}{3} \quad \frac{P}{n} = \text{const} \)
3.

\[ A. \quad \int_0^{\pi k} dx\, |\psi|^2 = 1 = \int_0^{\pi k} dx\, A^2 \left(1 - 2\cos(2kx) \cos^2 kx \right) \]

\[ 1 = \frac{A^2}{4} \left\{ \frac{\pi}{k} + \frac{\pi}{k^2} \right\} \quad \quad \frac{A^2 \cdot 3}{4k} \cdot \frac{1}{2} = 1 \quad A = \sqrt{\frac{8k}{3\pi}} \]

\[ B. \quad \mathcal{E}\psi = \frac{-\hbar^2}{2m} \frac{d^2\psi}{dx^2} + V\psi \quad V = \frac{(E + \frac{k^2\hbar^2}{2m})\psi}{\psi} \]

\[ \psi = A \sin^2 k\pi \quad \frac{d\psi}{dx} = 2Ak \sin k\pi \cos k\pi \]

\[ \frac{d^2\psi}{dx^2} = 2Ak^2 \left( \cos^2 k\pi - \sin^2 k\pi \right) = 2Ak^2 \left(1 - 2\sin^2 k\pi \right) \]

\[ V = E + \frac{\hbar^2 k^2}{2m} \left(1 - 2\sin^2 k\pi \right) = \left(E - \frac{\hbar^2 k^2}{2m} \right) + \frac{\hbar^2 k^2}{2m} \left(1 - \sin^2 k\pi \right) \]