1. (6 pts) A cubic block of wood 10.0 cm on a side, floats at the interface between oil and water with its lower surface 1.5 cm below the interface. The density of the oil is 790 kg/m$^3$, water can be assumed to have a density of 10$^3$ kg/m$^3$

(A) What is the pressure difference between the top surface of the oil and the bottom of the container? (2 pts) Give your answer in Pascal, 1 Pa = 1 N/m$^2$.

(B) What is the mass of the block? (3 pts)

(C) What would the mass of the block have to be reduced to by hollowing so that it would float on the oil? (1 pt)

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A) Container Bottom

\[ P = g(\rho_1 h_1 + \rho_2 h_2) = 1.75 \times 10^5 \text{ N/m}^2 \]

\[ \rho_1 = 790 \text{ kg/m}^3 \quad \rho_2 = 10^3 \text{ kg/m}^3 \quad h_1 = h_2 = 0.1 \quad g = 9.8 \text{ m/s}^2 \]

B) \[ P_{\text{top}} = \rho_1 g \cdot 0.15 = 1.18 \times 10^5 \text{ N/m}^2 \]

\[ P_{\text{bottom}} = \rho_1 g (0.1) + \rho_2 g (0.015) = 9.21 \times 10^5 \text{ N/m}^2 \]

Buoyancy Form \[ F_B = A(P_{\text{bottom}} - P_{\text{top}}) = 0.01 \text{m}^2 (9.21 - 1.18) \times 10^5 \text{ N} = 8.05 \text{ N} \]

\[ m = \frac{F_B}{g} = 0.82 \text{ kg} \]

C) Required \[ M \leq M_{\text{oil}} = (0.001) \cdot 790 \text{ kg/m}^3 = 0.79 \text{ kg} \]
2. (7 pts) A bead of mass $m$ can slide on a rotating hoop of radius $a$ in the presence of gravity. The hoop is rotating about a vertical axis as shown with angular velocity $\omega$. The position of the bead is defined by the angle $\beta$, and the bead can be considered to be a point particle. Give your answers in terms of the parameters: $m, a, g, \omega, \beta$.

(A) Assuming the bead slides without friction, what is the magnitude and direction of the force exerted by the hoop on the bead? (2 pts)

(B) For given values of the parameters $m, a, g,$ and $\omega,$ at what angles can the bead remain at a steady value of $\beta$? (2 pts)

(C) Find the critical rotation rate below which the bead can rest stably at the bottom of the hoop ($\beta = 0$). (2 pt)

(D) Now imagine the sense of rotation is reversed. How will your answers change? (1 pt)

\[ F_n \sin \beta = \frac{m v^2}{r} \]
\[ v = \omega r \]
\[ r = a \sin \beta \]

\[ |F_n| = m \omega^2 a \]

(B) Vertical Force Balance \[ F_n \cos \beta = mg \]
\[ \cos \beta = \frac{g}{\omega^2 a} \]
\[ \sin \beta = c \]

(C) if \[ |w| < \sqrt{g/a} \]
\[ \cos \beta = 1 \]
\[ \beta = 0 \]

(D) Sign of $w$ changes nothing
3. (7 pts) In the Bohr model of the hydrogen atom the electron is assumed to follow a classical circular orbit. The orbit is then quantized by requiring the circumference to be an integer $n$ de Broglie wavelengths.

(A) Show that in this model the radius of the $n$th state is given by $r_n = n^2a_0$. Derive an expression for $a_0$ in terms of the electron charge, electron mass, permittivity of free space, and Planck's constant. (3 pts)

(B) Show that the Energy of the $n$th state is given by

$$E_n = -\frac{1}{n^2} \frac{e^2}{8\pi \varepsilon_0 a_0},$$

(2 pts)

(C) If an electron makes a transition from state $n+1$ to state $n$ it emits a photon of what frequency? If $n$ is large how does this compare with the electron's classical frequency of rotation about the proton. (2 Pts)

(A) For circular orbit

$$\frac{mv^2}{r} = \frac{e^2}{4\pi \varepsilon_0 r^2} = \text{Coulomb Force}$$

Quantization $2\pi r_n = n\lambda$

$$\lambda = h/p$$

$$p_n = m v = \frac{nh}{2\pi r_n}$$

$$p_n^2 = \frac{me^2}{4\pi \varepsilon_0 r_n}$$

$$\frac{h^2 n^2}{(2\pi r_n)^2} = \frac{me^2}{4\pi \varepsilon_0 r_n}$$

$$r_n = n^2 a_0$$

$$a_0 = h^2 e_0 \frac{e^2}{nm e^2}$$

(B) Total Energy $= KE + PE = \frac{p^2}{2m} - \frac{e^2}{4\pi \varepsilon_0 r}$

$$E_n = -\frac{1}{n^2} \frac{e^2}{8\pi \varepsilon_0 a_0}$$

(C) $\frac{\omega \hbar}{2\pi} = E_{n+1} - E_n = \frac{e^2}{8\pi \varepsilon_0 a_0} \left( \frac{1}{n^2} \cdot \frac{1}{(n+1)^2} \right) = \frac{e^2}{4\pi \varepsilon_0 a_0} \frac{1}{n^3}$

$$\frac{\omega \hbar}{2\pi} = \frac{e^2}{4\pi \varepsilon_0 a_0} \frac{1}{n}$$

$$\frac{1}{n} = \frac{\lambda}{2\pi r} = \frac{h}{2\pi \varepsilon_0 r}$$

$$\frac{e^2}{4\pi \varepsilon_0 a_0} \frac{1}{n^3} = \frac{p^2}{m}$$

$$\omega = \frac{p}{m}$$