#1. (6 points) For the following circuit assume that the op-amp is ideal (linear, zero input currents and zero input difference voltage=vd=0).

![](image)

a) (3 points) Find the transfer function \( vo(s)/vi(s) \) when \( R1=R2=R3=R \) and give its zeros and poles.

b) (3 points) Discuss stability of the circuit, including conditions and nature.

#2 (7 points) The following circuit is described by the equations

\[
\begin{bmatrix} C & 0 \\ 0 & L \end{bmatrix} \frac{dx}{dt} = \begin{bmatrix} 0 & -1 \\ 1 & -R \end{bmatrix} x + \begin{bmatrix} 1 \\ 0 \end{bmatrix} \text{in}
\]

\[ v_0 = \begin{bmatrix} 0 & R \end{bmatrix} x \]

![](image)

a) (2 points) Give \( x \) in terms of voltage and current variables labelled in the circuit.

b) (3 points) Give the transfer function \( vo/\text{lin}(s) \).

c) (2 points) For \( C, L, R \) all positive, discuss if this is a low-pass, high-pass or band-pass circuit.
#3) (7 points) The following is a sectioned circuit diagram for a feedback circuit consisting of a
differential pair, an all-pass circuit, and a high-pass feedback circuit.
The differential pair is described by \( I_{\text{outDP}}/v_d = G_m \), the all-pass circuit is described by
\( v_{\text{outAP}}/I_{\text{inAP}}(s) = \frac{s-a}{s+a} \), and the high-pass feedback by \( v_{\text{outHP}}/v_{\text{inHP}}(s) = Cs \); all of \( C, G_m, \) and \( a \) are positive.

a) (3 points) Find the transfer function \( V_{\text{out}}/V_{\text{in}}(s) \).
b) (2 points) Show that there is a \( G_m \) for which this will be a sinusoidal oscillator; give the \( G_m \)
and oscillation frequency, \( f_{\text{osc}} \).
c) (2 points) If the output of the high-pass section were to become shorted, discuss how that will
affect a measurement of \( V_{\text{out}} \) in the laboratory.
#1. a) \[ V_o = \frac{V_o}{R_s + R_k} \]
\[ V_i = V_i - V_o = V_i - \frac{V_o}{R_s + R_k} \Rightarrow i = \frac{V_o}{R_s + R_k} \]
\[ i = i_c = i_R + i_C \]
\[ V_c = V_c - V_o = V_c - \frac{V_o}{R_s + R_k} \]
\[ \Rightarrow -\frac{V_c}{2} + \frac{V_o}{2} = V_c \Rightarrow \frac{V_o}{V_c} = \frac{2}{R_s + R_k} \]
\[ = \frac{V_o}{V_C} = \frac{V_o}{V_R} \]
\[ \Rightarrow \frac{1}{i} \rightarrow 0, \text{as } R \rightarrow 0 \]
\[ b) \text{If } R > 0, \text{ the circuit is unstable having a RHP pole, giving impulse response } \frac{V_o}{V_C} e^{-\frac{iC}{R}} \]
\[ \text{If } R = 0, \text{ the input is sent to a diode, with } V_o = 2V_c, \text{ giving stable behavior.} \]
\[ \text{Similarly, if } C = 0 \text{ or the input current is } 0, \text{ giving } \frac{V_o}{V_C} = 2 \Rightarrow \text{stable.} \]

2. Summing currents at the top of \( C \Rightarrow i_c + \frac{V_c}{R} + i_l \Rightarrow i_c + \frac{V_c}{R} = i_c + \frac{V_c}{R} \]
\[ \Rightarrow 1 \text{st leg } \Rightarrow i_c = i_l = i \]
\[ \text{Summing voltages around loops, } C - L - R \Rightarrow -V_l + i_l \frac{V_c}{R} + R \frac{V_c}{2} = 0 \Rightarrow \frac{V_c}{R} = \frac{V}{R} \]
\[ \Rightarrow \text{in 2nd leg } \Rightarrow i_c = i_l \]

3. a) \[ x = \frac{V_c}{V} \]

b) Taking Laplace transform \[ \begin{bmatrix} [2 & 0] \end{bmatrix} X(s) = \begin{bmatrix} 0 & -1 \end{bmatrix} X(s) + \begin{bmatrix} 1 & 1 \end{bmatrix} \]
\[ \begin{bmatrix} 0 & 0 \end{bmatrix} \]
\[ \Rightarrow X(s) = \begin{bmatrix} -1 & 1 \end{bmatrix} \begin{bmatrix} V & 0 \end{bmatrix} \]
\[ \Rightarrow X(s) = \begin{bmatrix} 0 & R \end{bmatrix} \begin{bmatrix} 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} 0 \end{bmatrix} \]
\[ \Rightarrow V_o = \frac{R}{1} \begin{bmatrix} 0 \end{bmatrix} \]

4. a) \[ \text{This is a low-pass circuit if } R, L, C \text{ all } > 0 \text{ as } \frac{\eta}{i} \rightarrow 0 \]

b) \[ \text{For } G_m c a = 1, \text{ which is } G_m c = 1 \Rightarrow \frac{V_m}{i_m} = \frac{G_m (a-\alpha)}{a-\alpha} \Rightarrow G_m c a = \frac{G_m (a-\alpha)}{a-\alpha} \]
\[ \Rightarrow \frac{V_m}{i_m} = \frac{G_m}{a-\alpha} \]
\[ \Rightarrow \text{as } a \rightarrow 0 \Rightarrow \text{poles and zero in } w \text{ axis} \]
\[ \Rightarrow G_m c a = 1 \Rightarrow \text{oscillation } \Rightarrow \alpha = \sqrt{\frac{G_m}{a}} \Rightarrow \text{as } a \rightarrow 0 \]
\[ c) \text{If high-pass output = short } \Rightarrow \text{no feedback } \Rightarrow V_o = V_{in} \text{ killing oscillation} \]
\[ \Rightarrow \frac{G_m}{V_{in}} \text{ for } c = 0 \Rightarrow \text{measure only "damped" response, no self oscillation} \]