Devices

A helpful equation: \[
\frac{dE(x)}{dx} = \frac{\rho(x)}{\varepsilon_s}
\]

Problem #1 – (6 pts) Diode Basics
Consider a silicon diode with the charge density distribution illustrated on the right. Make LARGE labeled sketches of:

a) the resulting electric field \( E(x) \). (Be sure to label the field at all relevant transitions.)

b) the potential \( \phi(x) \). For this plot, assume that \( \phi(0) = 0 \).

Problem #2 – (7 pts) Diode Breakdown
In a reverse-biased pn-junction diode, there is a critical electric field, \( E_{\text{crit}} \), at which breakdown occurs (whether due to a tunneling or an avalanche mechanism). Given the depletion region width equation

\[
W_{\text{depletion}} = \sqrt{\frac{2\varepsilon_s}{q} \left( \frac{1}{N_A} + \frac{1}{N_D} \right) V_0}
\]
solve for the applied reverse bias voltage \( V_R \) where breakdown occurs.

\( V_0 \) represents the ‘built-in voltage’, \( N_A \) and \( N_D \) represent the doping concentrations in \( p \) and \( n \) regions, and \( \varepsilon_s \) represents the permittivity of silicon.

Problem #3 – (7 pts) n-channel Enhancement MOSFET in triode
Occasionally it becomes important to model the small signal operation of a MOSFET in the triode (aka ‘ohmic’) mode of operation. Using the triode equation given below, solve for the transconductance \( g_m \) and drain resistance \( r_d \) that can be used to model the transistor at a fixed operating point where \( v_{GS} - V_{th} > v_{DS} > 0 \). The two parameters \( g_m \) and \( r_d \) capture the effect of the gate and drain voltages on the drain current.

\[
i_D = k_n \frac{W}{L} \left( v_{GS} - V_{th} \right) v_{DS} - \frac{1}{2} v_{DS}^2 \right] \quad \text{where } k_n', W, L, \text{ and } V_{th}
\]

\( V_{th} \) are the constant parameters: process transconductance, channel width, channel length, and threshold voltage, respectively.
Problem #1
The slope of the electric field is proportional to the charge density and so we have a linearly decreasing electric field where the charge density is fixed and negative \((0 < x < x_1)\), followed by a zero slope in the electric field, and terminated by a linearly rising region where the charge density is fixed and positive \((x_2 < x < x_3)\). In a diode, the depletion charge balance each other and so the electric field well away from the junction is zero.

\[
E(x) = \begin{cases} 
-qN_Ax_1 \frac{x}{\varepsilon_S} & \text{for } 0 < x < x_1 \\
0 & \text{for } x_1 < x < x_2 \\
qN_Dx_2 \frac{x - x_2}{\varepsilon_S} & \text{for } x_2 < x < x_3 
\end{cases}
\]

The linearly decreasing and increasing regions of the electric field \((0 < x < x_1 \text{ and } x_2 < x < x_3)\) produce quadratically-shaped regions in the potential plot, whereas the constant electric field region \((x_1 < x < x_2)\) produces a linear region in the potential.

Problem #2 – Diode Breakdown
In a reverse-biased pn-junction diode, there is a critical electric field, \(E_{\text{crit}}\), at which breakdown occurs, whether due to a tunneling or an avalanche mechanism. Given the depletion region width equation \(W_{\text{depletion}} = \sqrt{\frac{2\varepsilon}{q} \left( \frac{1}{N_A} + \frac{1}{N_D} \right) V_0} \) (where \(V_0\) represents the ‘built-in voltage’), solve for the applied reverse bias voltage \(V_R\) where breakdown occurs.

To solve this problem, there are several things that must be known: 1) \(x_aN_D = x_pN_A\) due to charge balance, 2) that the peak electric field occurs at the junction and is:
\[ |E_{\text{peak}}| = x_p \frac{q N_A}{\varepsilon_S} = x_n \frac{q N_D}{\varepsilon_S} \] (where \( x_p \) and \( x_n \) represent the depletion region widths on each side of the junction), and 3) that the reverse-bias voltage \( V_R \) is added to the \( V_0 \) in the width equation.

\[ x_n N_D = x_p N_A \] gives us: \( x_p = x_n \frac{N_D}{N_A} \), so \( W_{\text{depletion}} = x_n + x_p = x_n + x_n \frac{N_D}{N_A} = x_n \left( \frac{N_A + N_D}{N_A} \right) \)

\[ x_n \left( \frac{N_A + N_D}{N_A} \right) = \sqrt{\frac{2 \varepsilon_s}{q} \left( \frac{1}{N_A} + \frac{1}{N_D} \right)} \left( V_0 + V_R \right) \Rightarrow x_n = \sqrt{\frac{2 \varepsilon_s}{q} \left( \frac{N_A}{(N_D + N_A) N_D} \right)} \left( V_0 + V_R \right) \]

To find what \( V_R \) produces an \( |E_{\text{peak}}| = E_{\text{crit}} \), we solve for \( V_R \) where,

\[ E_{\text{crit}} = x_n \frac{q N_D}{\varepsilon_S} = q N_D \sqrt{\frac{2 \varepsilon_s}{q} \left( \frac{N_A}{(N_D + N_A) N_D} \right)} \left( V_0 + V_R \right) \]

\[
\frac{E_{\text{crit}}}{2q} \frac{\varepsilon_s}{N_D N_A} N_D + N_A - V_0 = V_R
\]

**Problem #3**

The dependent current source represents how the gate voltage affects the drain current \((\partial i_D / \partial V_{GS})\) and the drain resistance represents how the drain voltage affects the drain current \((\partial i_D / \partial V_{DS})\).

\[ i_D = k_n \frac{W}{L} \left[ (V_{GS} - V_t) V_{DS} - \frac{1}{2} V_{DS}^2 \right] \]

\[ g_m = \frac{\partial i_D}{\partial V_{GS}} = k_n \frac{W}{L} V_{DS} \]

and

\[ g_d = \frac{\partial i_D}{\partial V_{DS}} = k_n \frac{W}{L} \left( (V_{GS} - V_t) - V_{DS} \right) \]

so,

\[ r_d = \frac{1}{g_d} = \frac{1}{k_n \frac{W}{L} \left( (V_{GS} - V_t) - V_{DS} \right)} \]