Circuits

1. 7 points

For the following circuit the op-amp is ideal.

a) Set up the differential equation for $v_o(t)$ versus $v_i(t)$.

b) Give the voltage transfer function $V_o(s)/V_i(s)$.

c) Give the impulse response, $v_o(t)$ for $v_i(t) = \delta(t)$.

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\[ \begin{array}{c}
R \\
L \\
C \\
\hline
\end{array} \]
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2. 7 points

The following circuit has $i_D = 10^{-3}(v_{GS} - 1)^2$, $V_{DD} = 8V$, $V_G = 5V$ and $v_i$ much smaller than 5V in magnitude.

a) Give the value of $i_D$ for $v_i = 0$.

b) Find the range of values of $R$ such that $V_0 \leq V_0 \leq 1$ (assuming $v_i = 0$).

c) Use $V_{DD}$ and two resistors, $R_1$ & $R_2$, to replace $V_G$. Draw the new circuit and give some values for $R_1$ & $R_2$; show how $v_i$ is coupled in.

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\[ \begin{array}{c}
\text{VG} \\
\text{5Vdc} \\
\hline
\end{array} \]
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3. 6 points

For the following circuit the diodes are ideal ($i = 0$ for $v < 0$ and $v = 0$ for $i > 0$). Assume that $v_i(t) = 2\sin(2t)$ for $0 \leq t$ and that at $t = 0$ the equal positive capacitors are uncharged.

a) Give $v_C(0+)$.  

b) Sketch $v_C(t)$ and $v_o(t)$ for two periods of $v_i(t)$. 

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\[ \begin{array}{c}
\text{VI} \\
\hline
\end{array} \]
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1. \( \text{at the input voltage of the op. amp } I = 0 \), \( v_i = R_i R + L R_i I d x \) and \( C I d v_i = -I \) (both currents, left to right) and at the input current to the op. amp \( I = 0 \) \( \Rightarrow i_R = i_C \Rightarrow v_C = -RC \frac{d q}{dt} = L \frac{d}{dt} \left( C \frac{d v_i}{dt} \right) \)

(a) \( \Rightarrow L \frac{d^2 v_C(t)}{dt^2} + RC \frac{d v_C(t)}{dt} = -v_i \)

Let \( v_i = v(t) = \left( L \frac{d^2}{dt^2} + RC \frac{d}{dt} \right) v(t) = -v(t) \)

(b) \( \Rightarrow v_C(t) = \frac{\frac{1}{L} \frac{d q}{dt}}{v_C(t)} = \frac{-1}{C (L + R)} \)

(c) \( \text{Then } v_1(t) = \delta(t), \ v_i(t) = 1 \), \( v_C(t) = \frac{\frac{1}{L} \frac{d q}{dt}}{v_C(t)} = \frac{-1}{R C} \frac{1}{1(t)} + \frac{1}{R C} E 1(t) \) where \( 1(t) = \text{unit step} = \begin{cases} 0 & t < 0 \\ 1 & t > 0 \end{cases} \)

\#2 \( \text{So that } v_i = 0, \ v_0 = V_0 \Rightarrow v_i = 10 \times 10^{-3} (5 - 1) = 16 \times 10^{-3} \)

(a) \( \Rightarrow i_d = 16 \text{ mA} \)

\( v_0 = V_0 - R i_d = 8 - R \times 16 \times 10^{-3} \Rightarrow v_0 = 4 = 7 - R \times 16 \times 10^{-3} \geq 4 \)

(b) \( \Rightarrow R \leq \frac{1}{16 \times 10^{-3}} = 0.625 \times 10^3 \text{ Ohm} \leq 250 \text{ Ohm} \), \( 0 < R \leq 250 \text{ Ohm} \)

(c) \( C_c \text{ large } \Rightarrow \text{allows } v_C \text{ to connect} \)

\( \text{voltage divides} \)

\( v_C = \frac{R_2}{R_1 + R_2} \frac{1}{v_{DD}} \)

\( \Rightarrow \frac{1 + R_2}{R_1} = \frac{R_2}{R_1} = \frac{8}{5} - 1 = 3/5 \)

\( \Rightarrow R_2 = \frac{3}{5} R_1 \). Choose \( R_1 \) large, such as 5 M\text{ Ohm} \Rightarrow R_2 = 3 \text{ M\text{ Ohm}}

\#3. \( \text{at } t = t_0, \ v_C(t_0) = 2 \sin t_0 = 0 \) appears \( v_i(t) \) after \( t_0 \) as well

(a) \( v_C(t_0) = 0 \)

(b) \( \text{The 1st half cycle } v_i(t) = v(t) \Rightarrow \frac{d}{dt} v_C(t) = 2 \sin 2t, 0 \leq t \leq t_0 \frac{1}{2} \)

Then, since the capacitors are charged and can not discharge

\( v_C(t) = 2, 2t > t_0 \frac{1}{2} \)

\( \Rightarrow \text{2nd cycle at } t = t_0 \frac{1}{2} \)