
ENEE 359a
Lecture/s 3-5
Transistors &
CMOS Inverter

Bruce Jacob

University of
Maryland
ECE Dept.

SLIDE 1

ENEE 359a

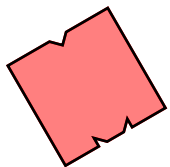
Digital VLSI Circuits

P/N Junction, MOS Transistors, CMOS Inverter

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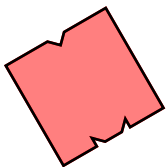
Credit where credit is due:

Slides contain original artwork (© Jacob 2004) as well as material taken liberally from Irwin & Vijay's CSE477 slides (PSU), Schmit & Strojwas's 18-322 slides (CMU), Wolf's slides for *Modern VLSI Design*, and/or Rabaey's slides (UCB).
Device physics: <http://hyperphysics.phy-astr.gsu.edu/hbase/solids/sselcn.html>



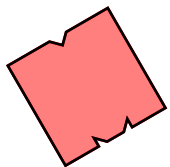
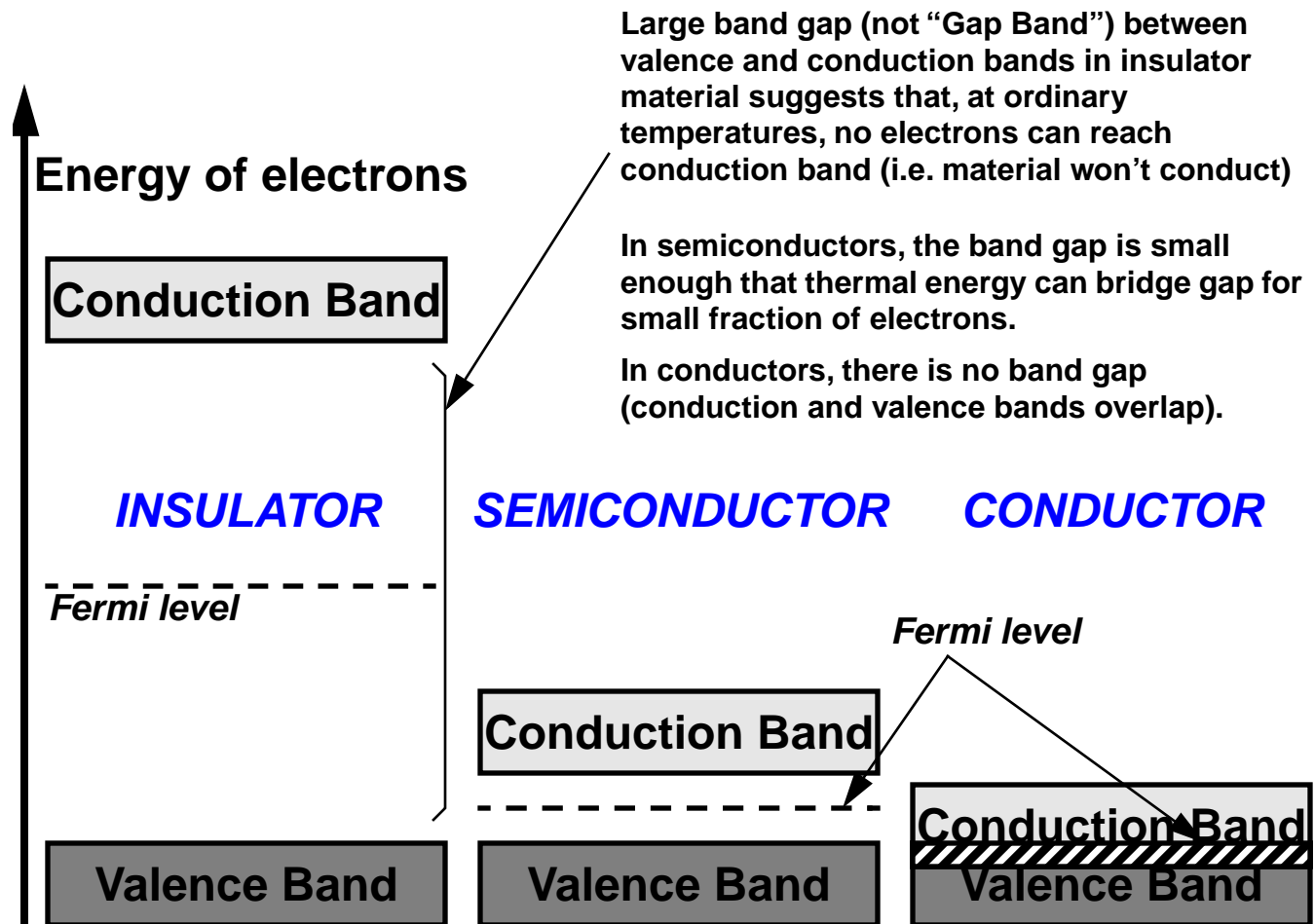
Overview

- **Electrons & holes, bands & band gaps, insulators, conductors, semiconductors**
- **Silicon crystal lattice & doping**
- **P/N junction & parasitic capacitance**
- **n-type/n-channel MOSFET**
- **Timing analysis of MOSFET, capacitance**
- **Body effect, series-connected FETs**
- **CMOS inverter: timing, switching threshold, transistor sizing**
- **Dynamic behavior (preview)**

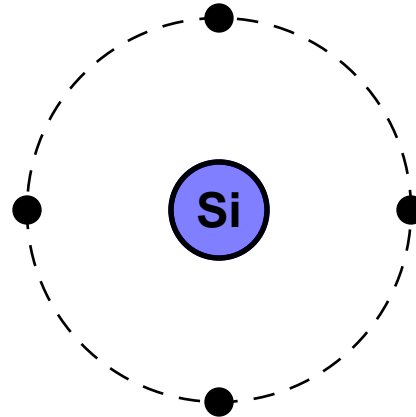


What Is Conductivity?

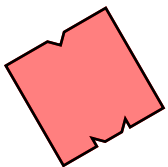
Perspective from Band Theory of Solids:



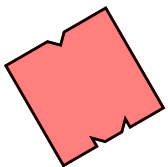
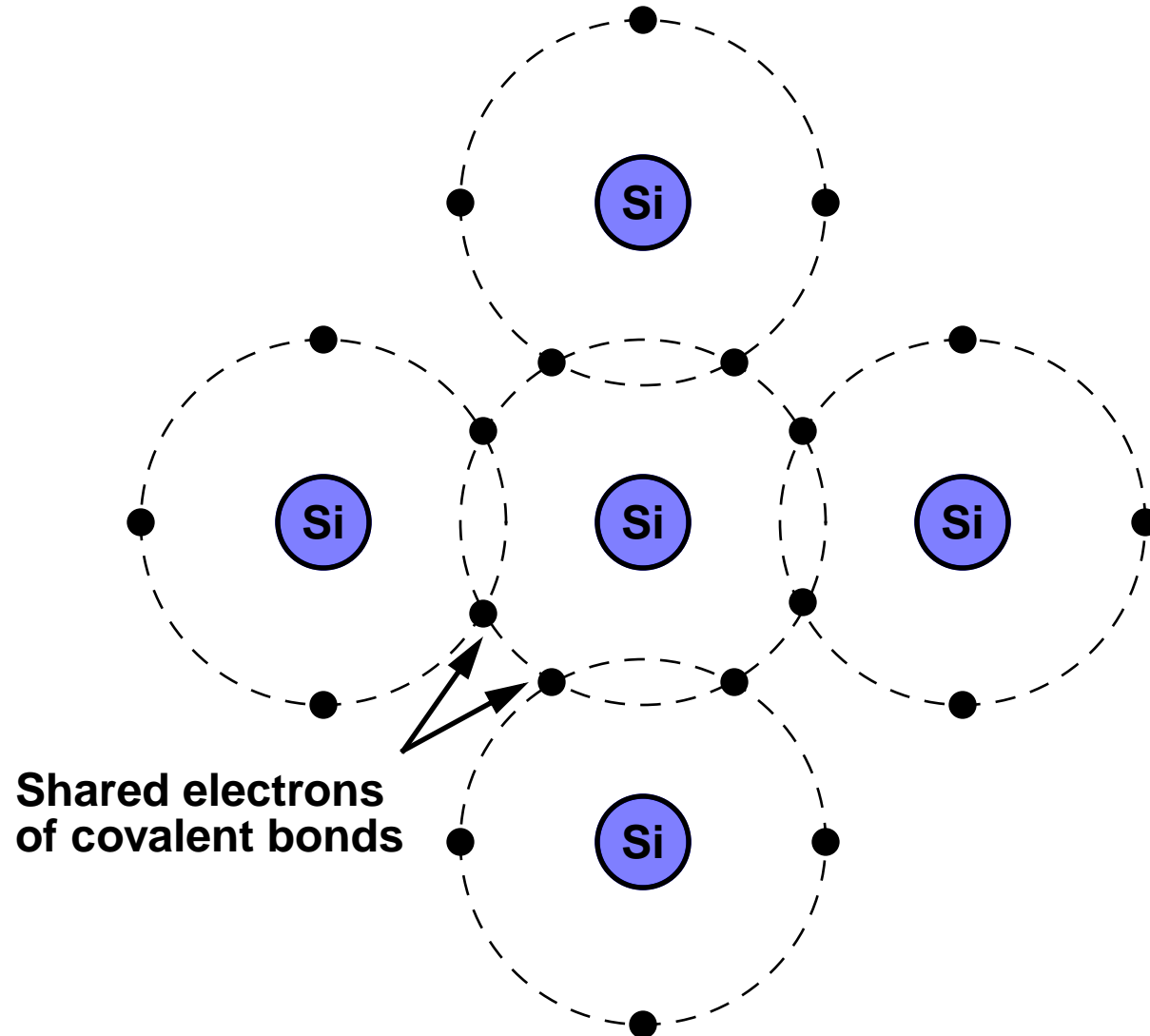
Silicon, Specifically



14 protons in nucleus
4 valence electrons

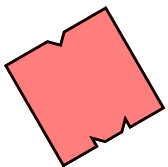
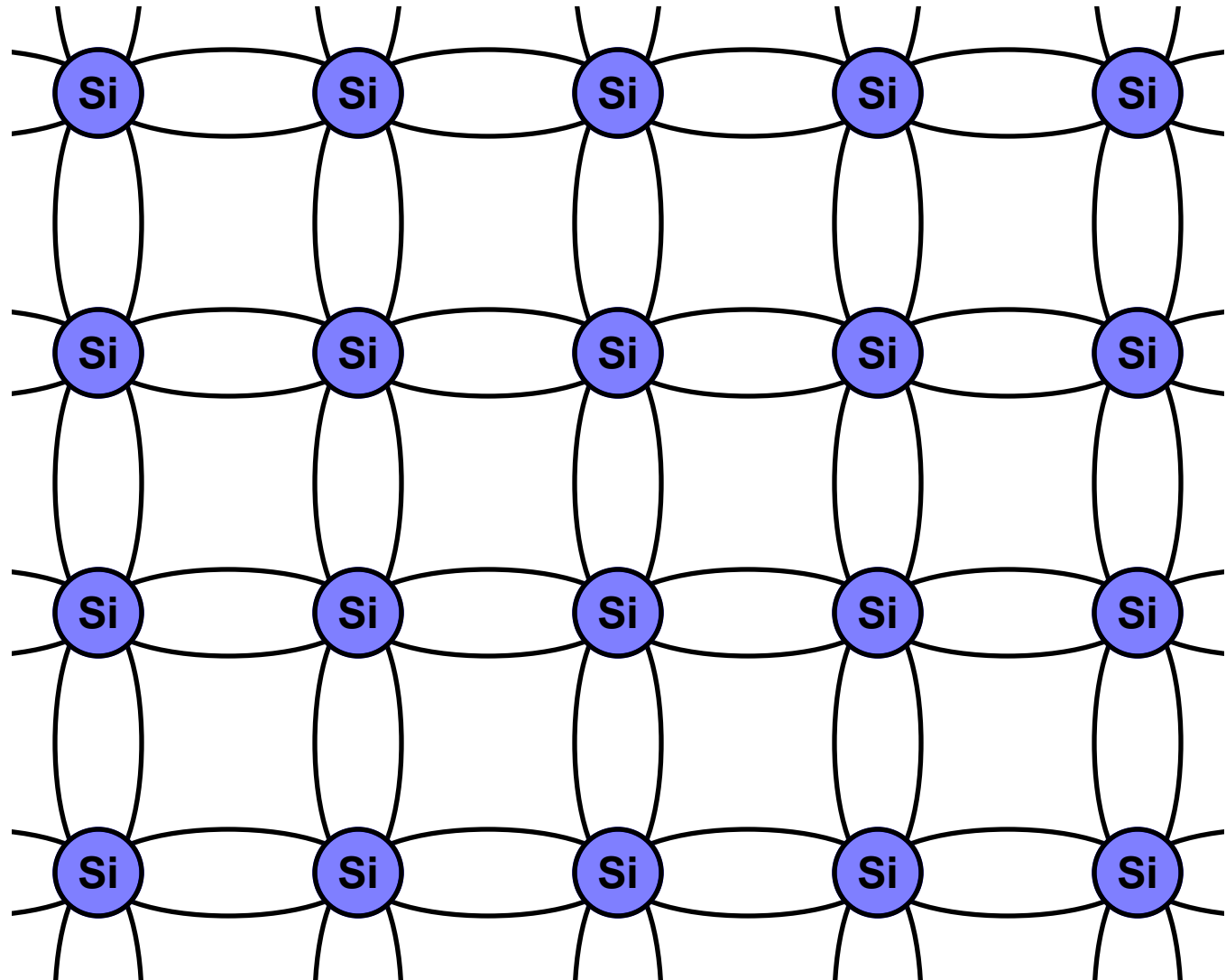


Silicon, Specifically



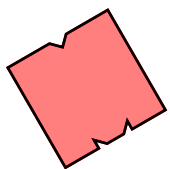
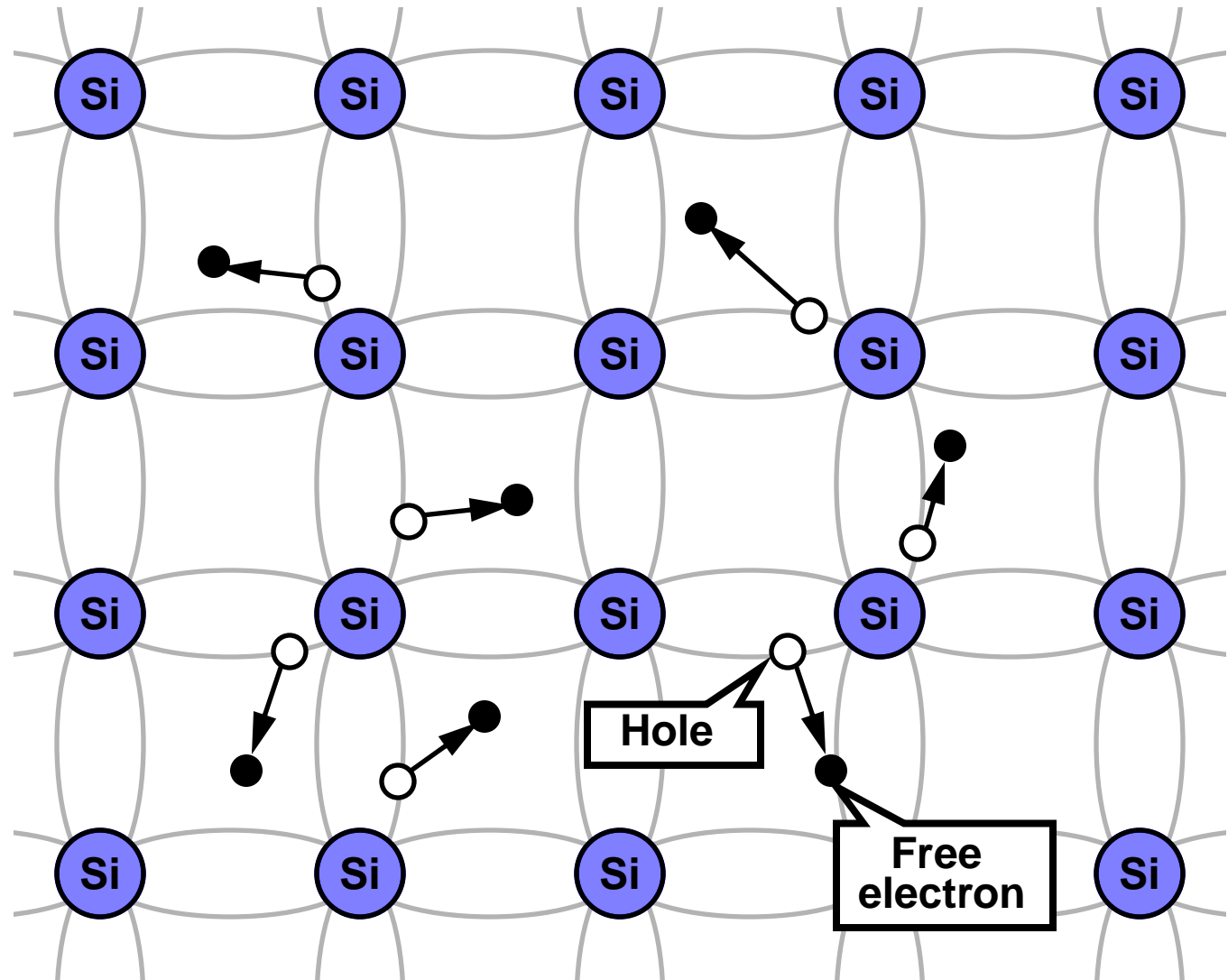
Silicon, Specifically

Silicon Lattice (artistic license exploited)



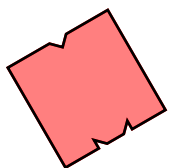
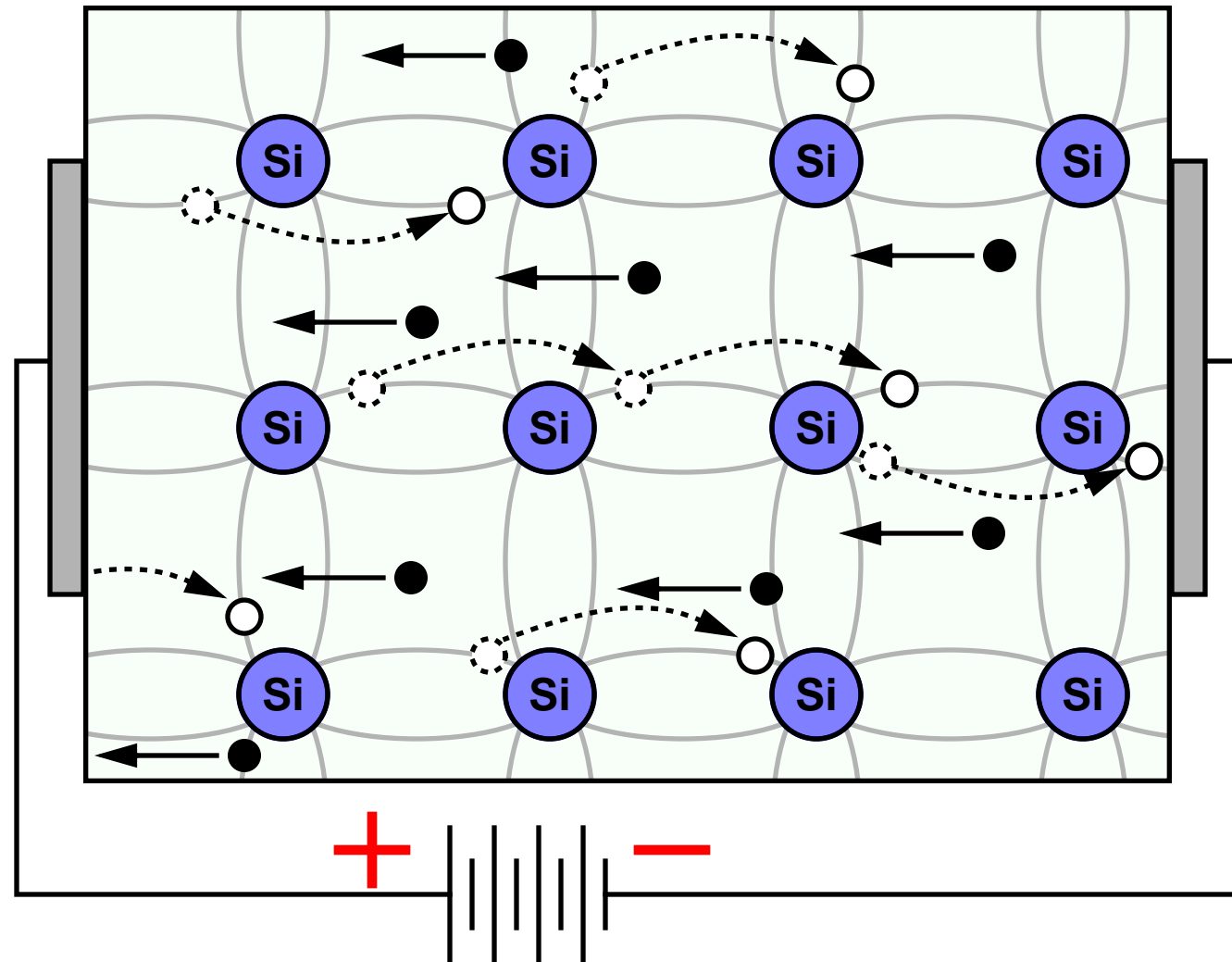
Silicon, Specifically

Silicon Lattice — It *is* a semiconductor



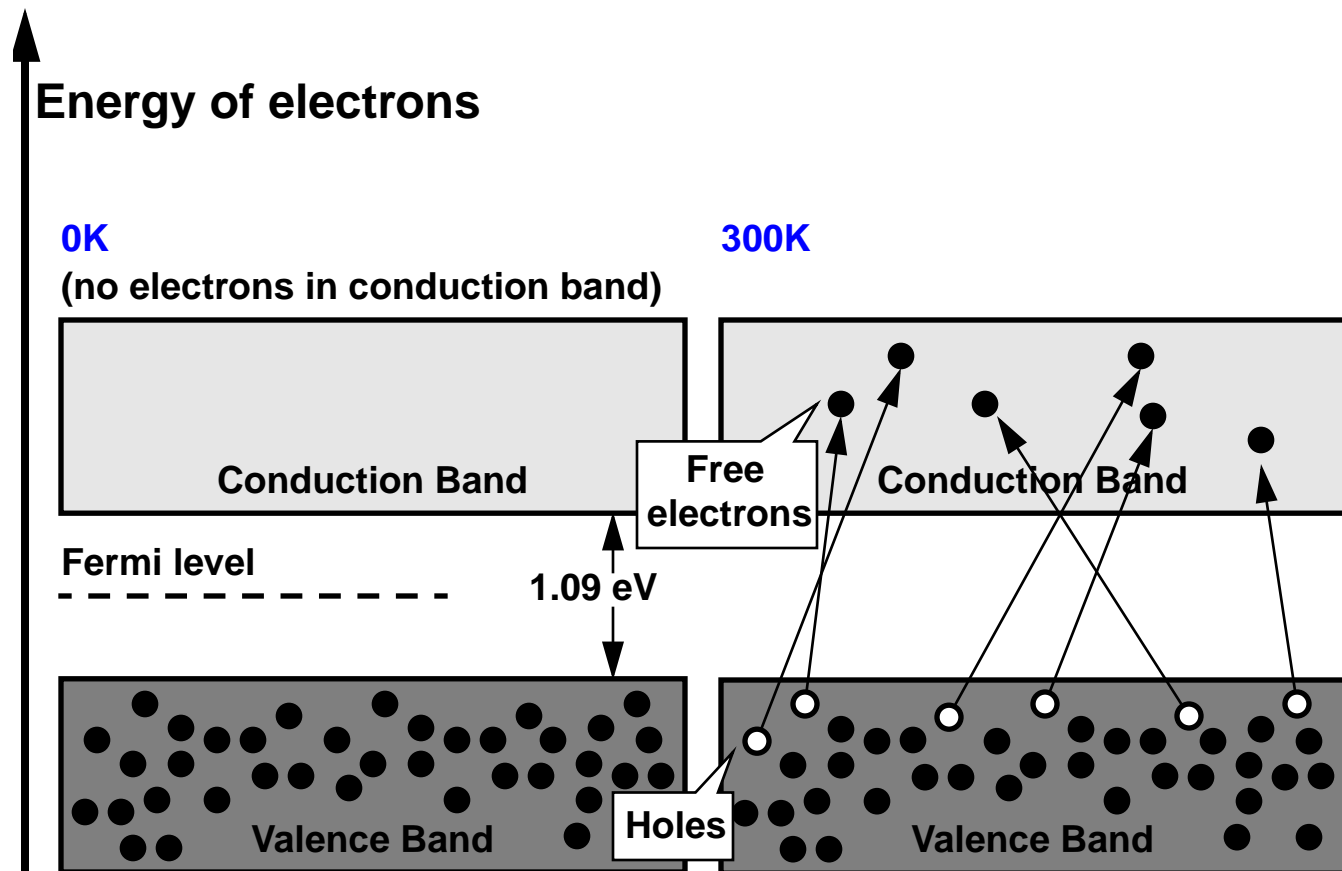
Silicon, Specifically

Semiconductor current: electron/hole flow

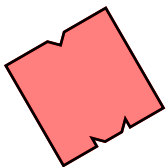


Silicon, Specifically

Perspective from Band Theory of Solids:

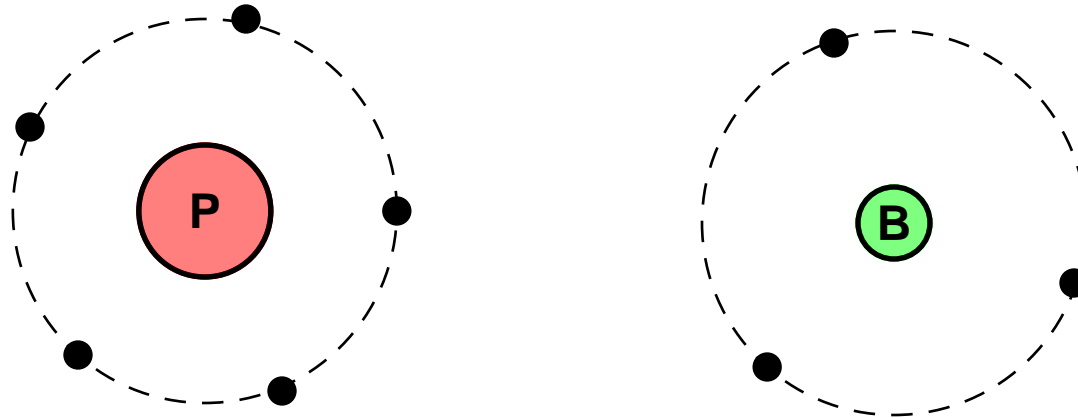


- **Conductivity is non-zero; mobile electrons/holes in conduction/valence band; can be increased w/ doping**



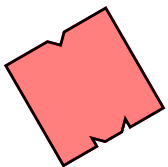
Silicon, Specifically

Doping: small % of foreign atoms in lattice

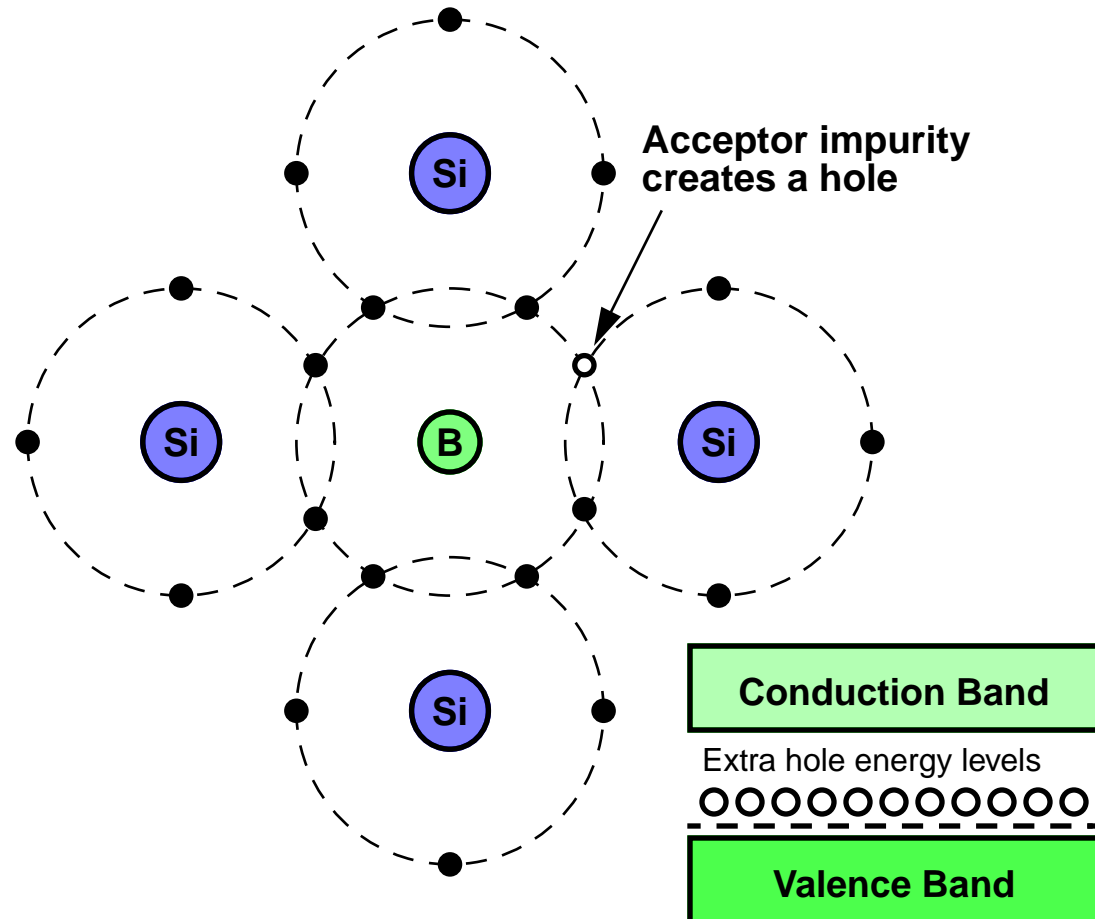


Breaks up regular lattice, produces dramatic changes in electrical properties

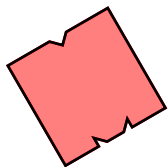
- **Donors:** pentavalent impurities (5 valence electrons) produce **n-type** semiconductors by adding electrons. E.g. **antimony, arsenic, phosphorus**
- **Acceptors:** trivalent impurities (3 valence electrons) produce **p-type** semiconductors by adding electron deficiencies (“holes”). E.g. **boron, aluminum, gallium**



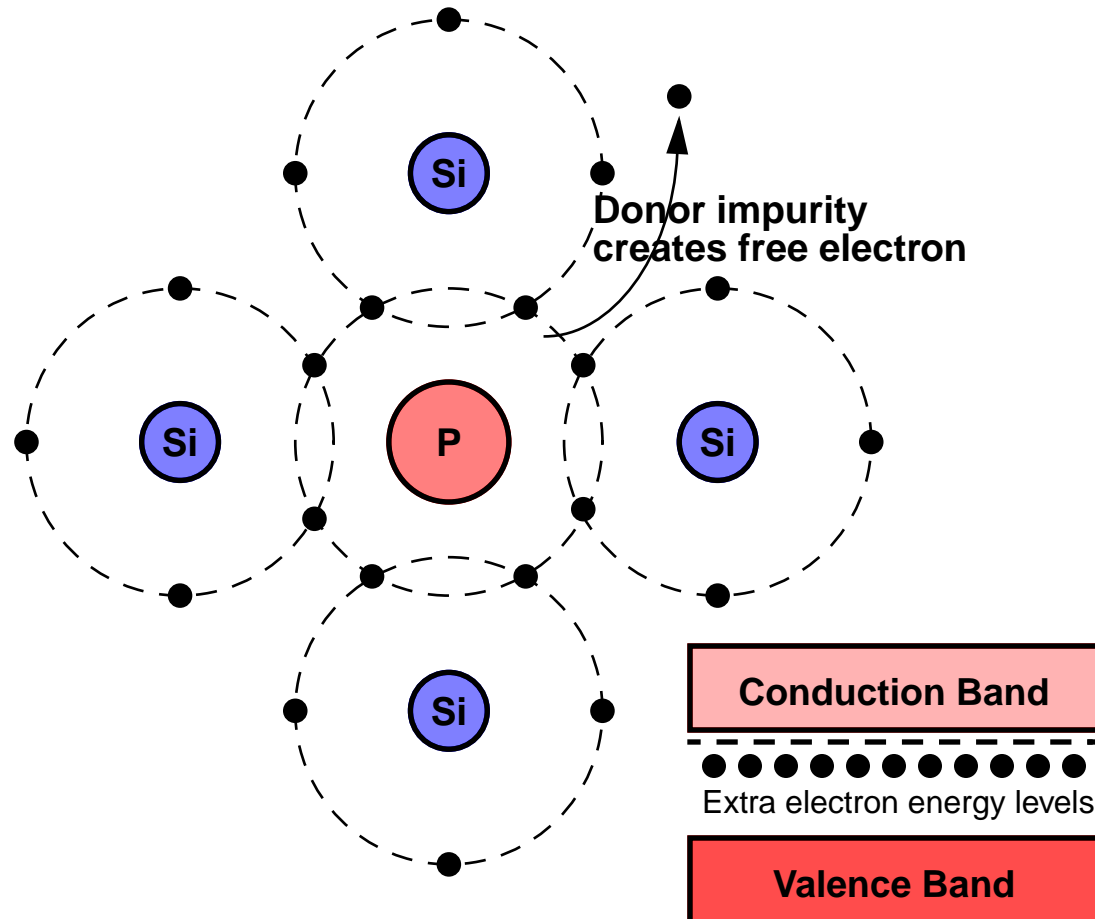
P-Type Semiconductor



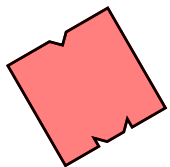
Addition of acceptor impurities contributes hole energy levels low in the semiconductor band gap so that electrons can be easily excited from the valence band into these levels, leaving mobile holes in the valence band. This shifts the effective Fermi level to a point about halfway between the acceptor levels and the valence band. Electrons can be elevated from the valence band to the holes in the band gap with the energy provided by an applied voltage. Since electrons can be exchanged between the holes, the holes are said to be mobile. Holes are said to be the "majority carriers" for current flow in a p-type semiconductor.



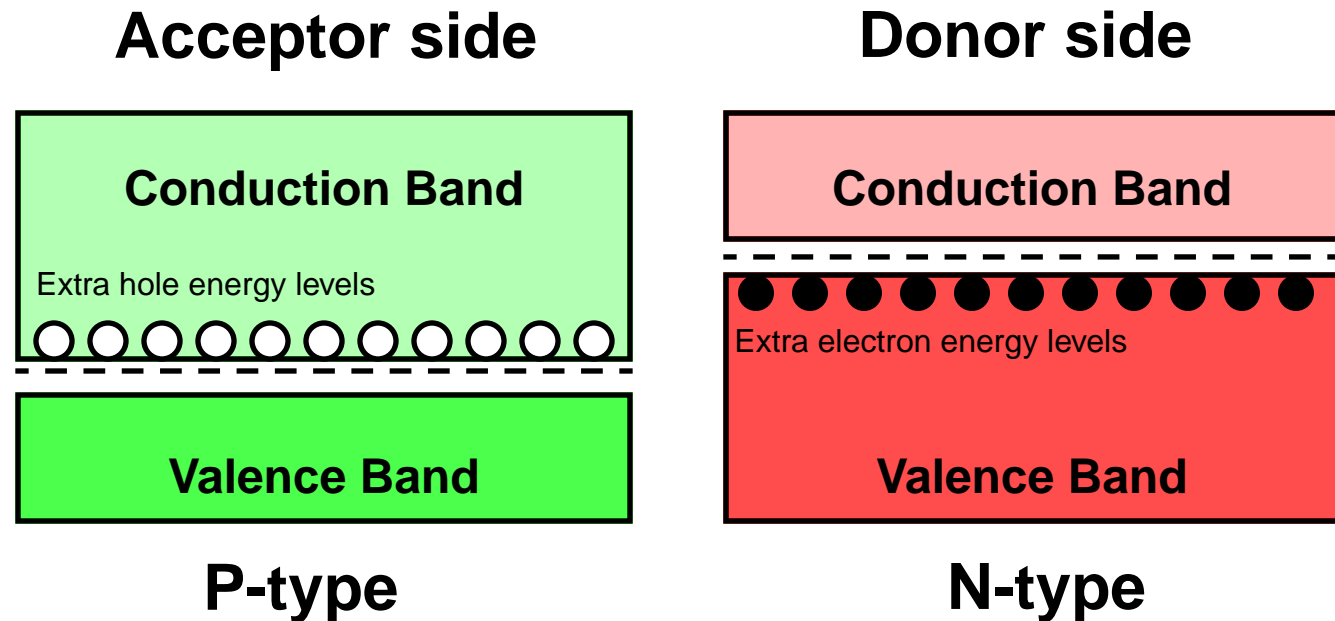
N-Type Semiconductor



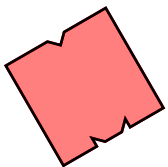
Addition of donor impurities contributes electron energy levels high in the semiconductor band gap so that electrons can be easily excited into the conduction band. This shifts the Fermi level to a point about halfway between the donor levels and the conduction band. Electrons can be elevated to the conduction band with the energy provided by an applied voltage and move through the material. Electrons are said to be the "majority carriers" for current flow in an n-type semiconductor.



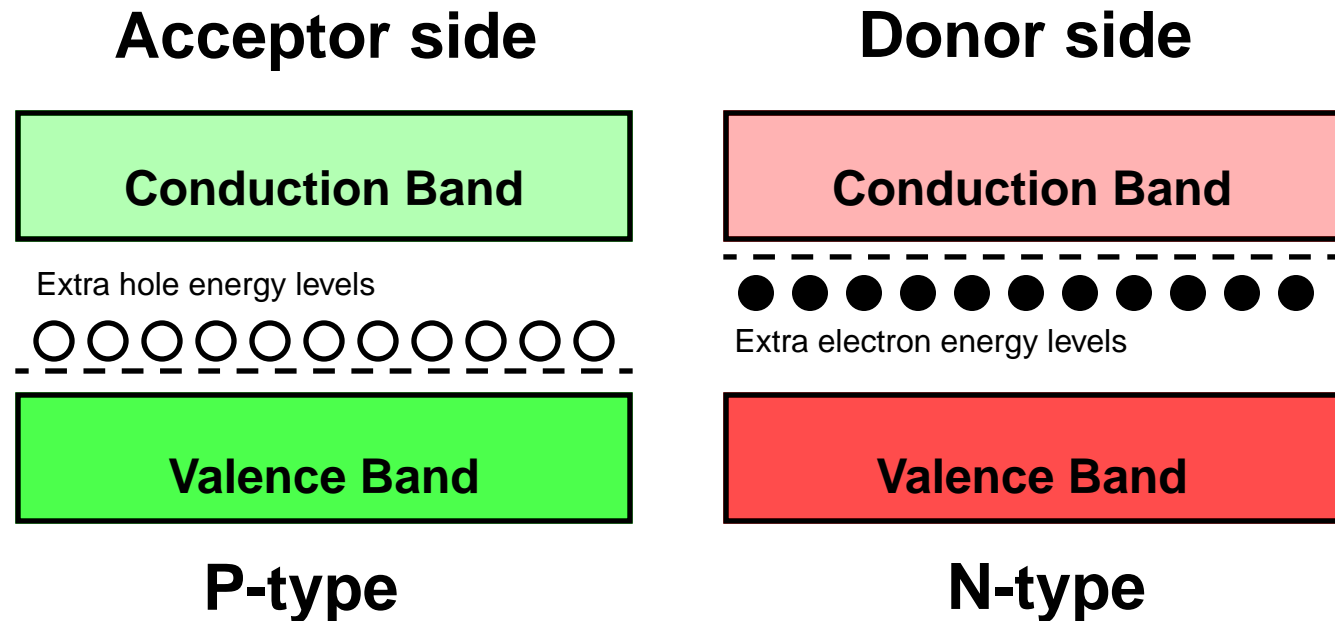
One Way to Think About It



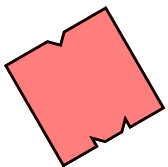
- **P-type:** Conduction band is pulled down close to the valence band by the creation of available holes (willing acceptors of free electrons)
- **N-type:** Valence band is pushed up close to the conduction band by the addition of mobile electrons



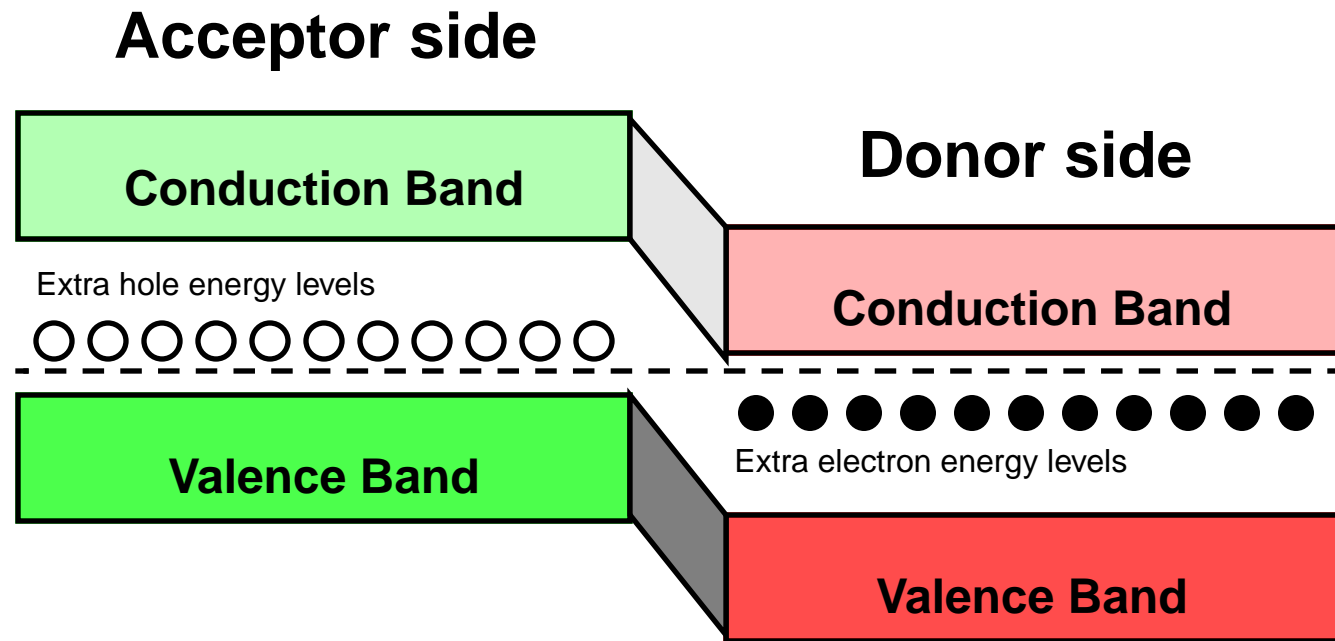
The P/N Junction



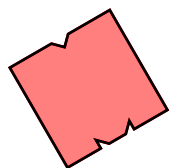
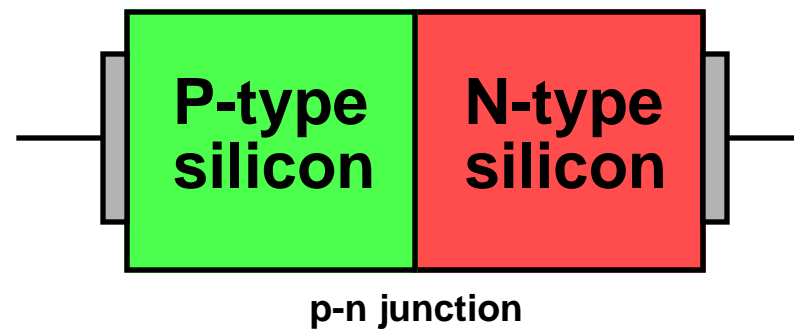
- **P-type:** extra holes in band gap allow excitation of valence-band electrons, leaving mobile holes in valence band
- **N-type:** electron energy levels near the top of the band gap allow easy excitation of electrons into conduction band



The P/N Junction

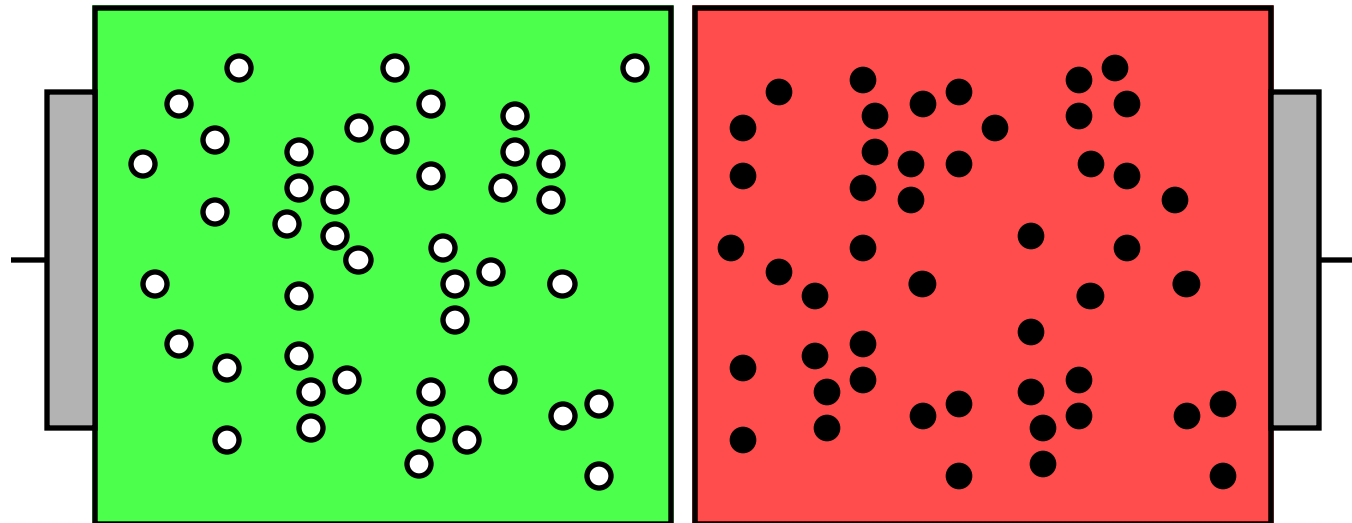


Diode

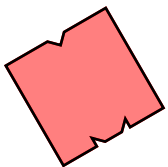


The P/N Junction

DEPLETION REGION

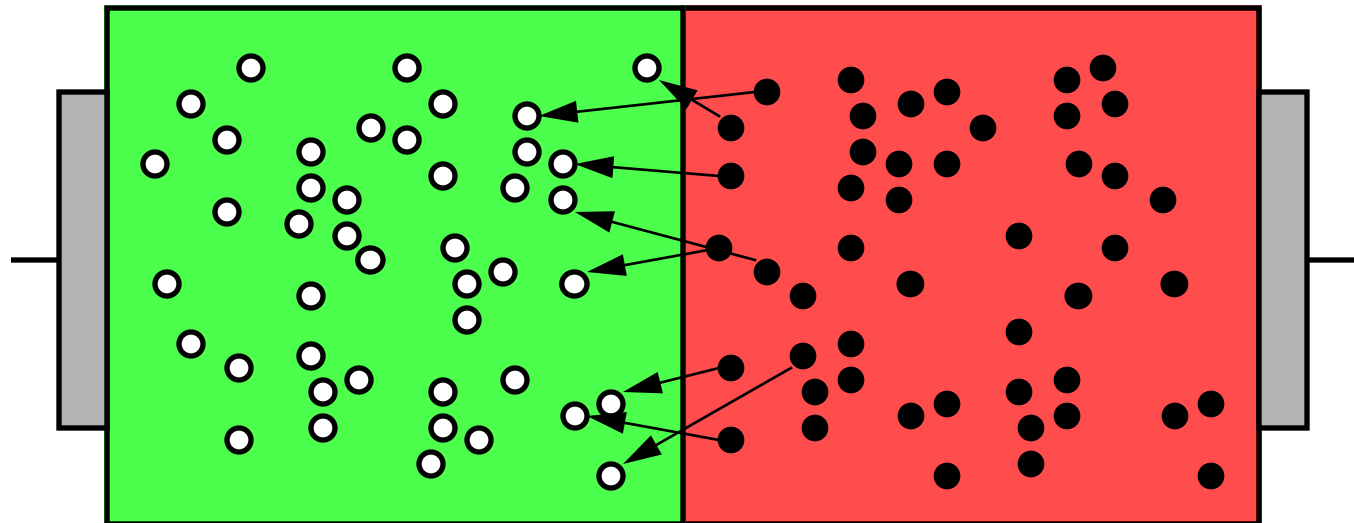


If not touching, nothing happens



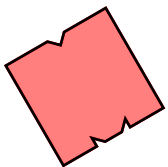
The P/N Junction

DEPLETION REGION



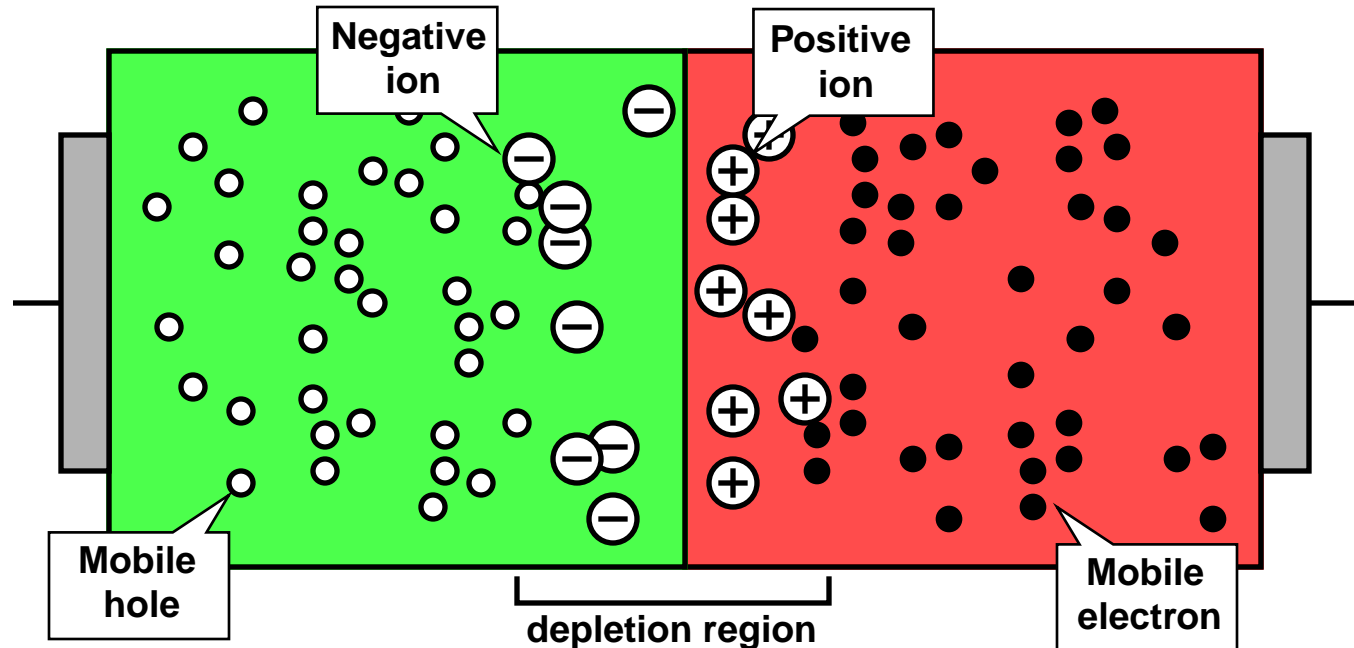
**With a connection, electrons from n-region
in conduction band diffuse across junction
and combine with holes in p-region**

(why doesn't this continue indefinitely?)

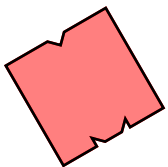


The P/N Junction

DEPLETION REGION

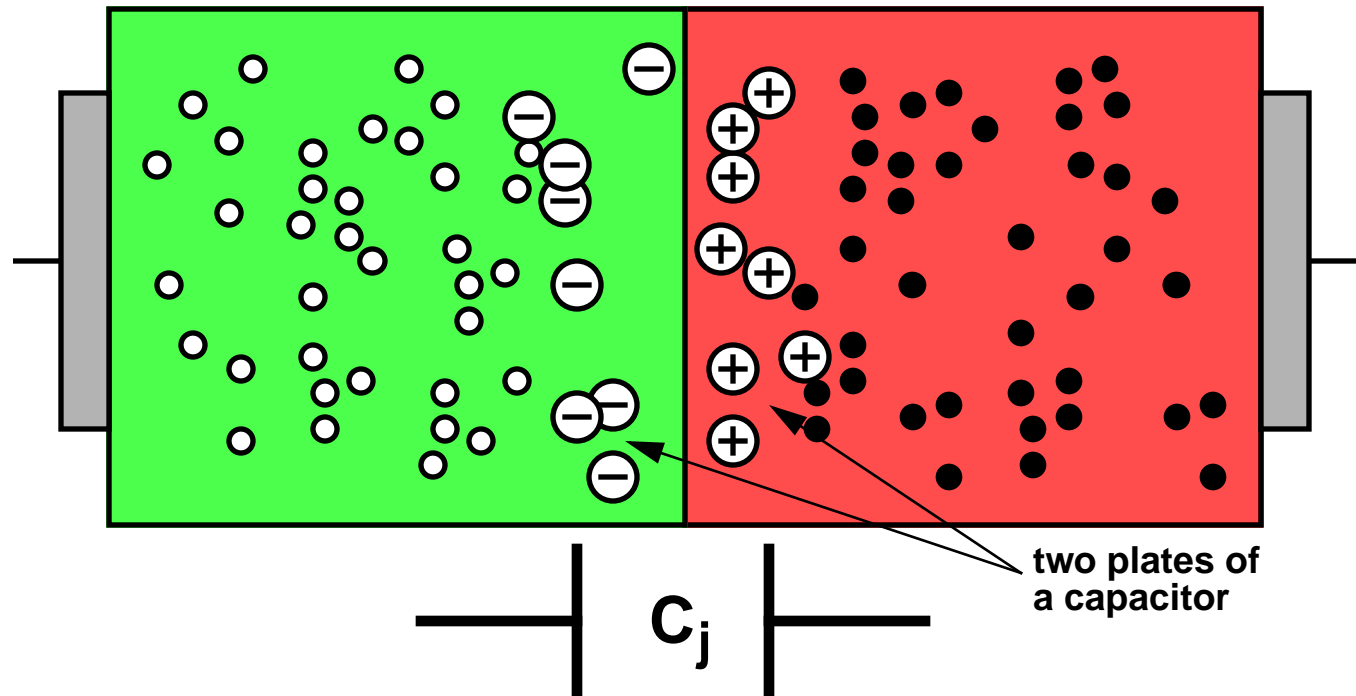


Ions are formed on both sides of junction (negative ion from filled hole; positive ion from removed electron). This forms a space charge that impedes further electron flow.



The P/N Junction

DEPLETION REGION

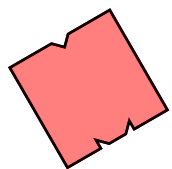


Parasitic capacitance:

$$C_j = \frac{C_{j0}}{\left[1 - \frac{V_d}{\phi_0}\right]^m}$$

Built-in junction potential:

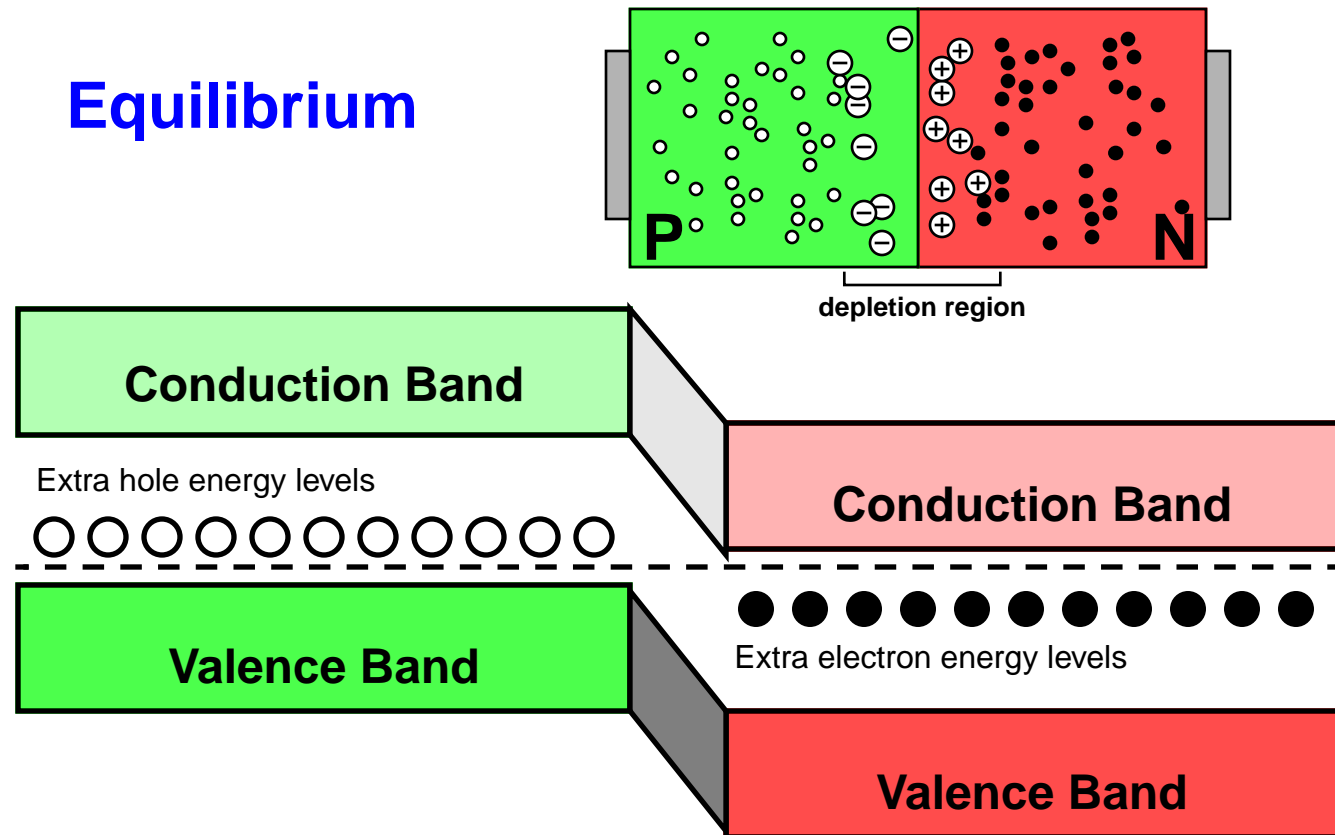
$$\phi_0 = \phi_T \cdot \ln\left(\frac{N_A N_D}{n_i^2}\right)$$



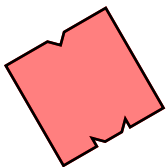
The P/N Junction

BIAS EFFECT on DEPLETION REGION

Equilibrium

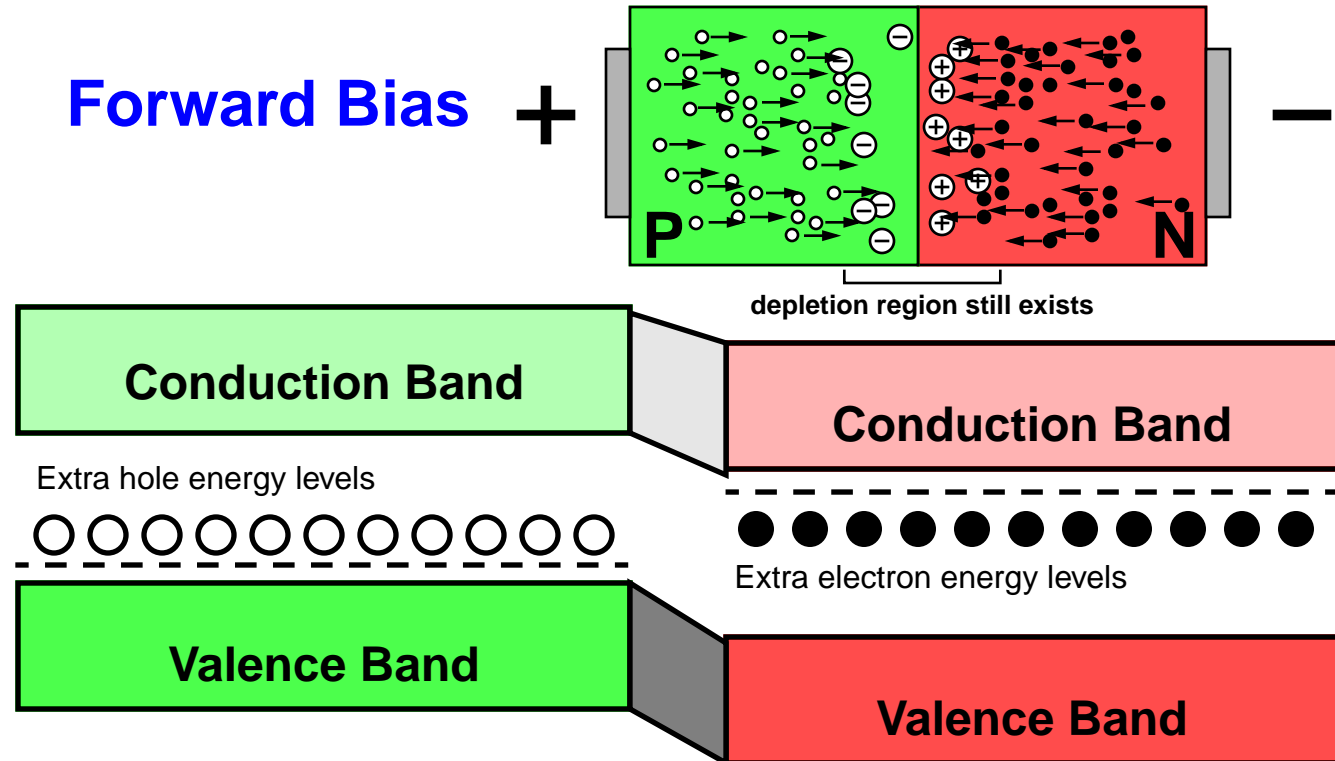


- Upward = increased electron energy (must supply energy to make electron go up or hole to go down)
- Drift-diffusion equilibrium (current *is* flowing)

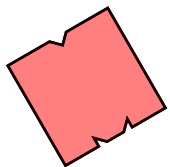


The P/N Junction

BIAS EFFECT on DEPLETION REGION

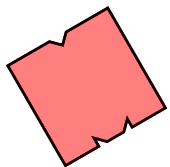
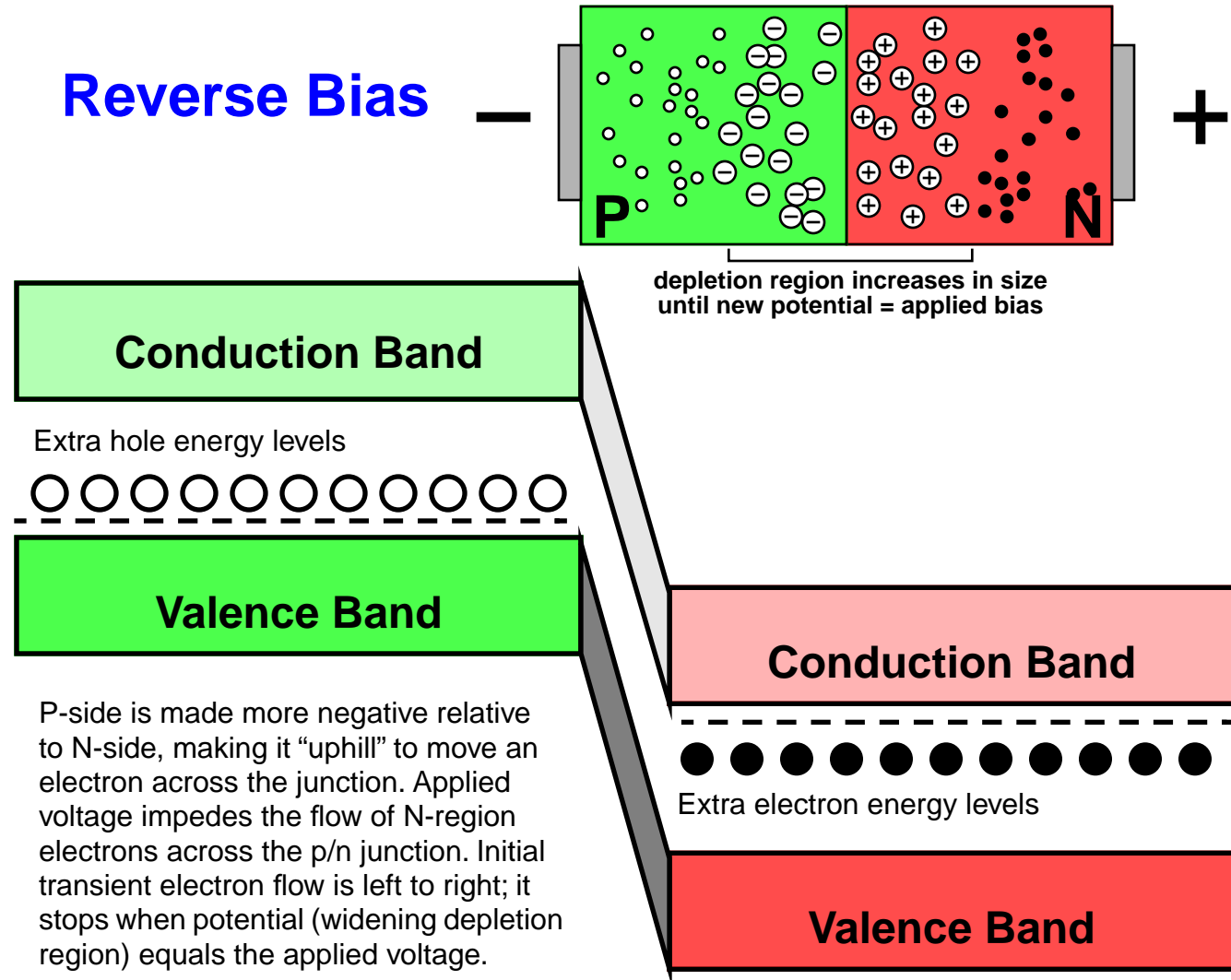


P-side is made more positive relative to N-side, making it “downhill” to move an electron across the junction. Electron on N-side can fill a vacancy (“hole”) on P-side & move from hole to hole to the left to positive terminal (hole “moves” right).



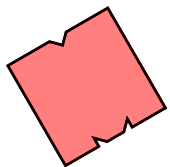
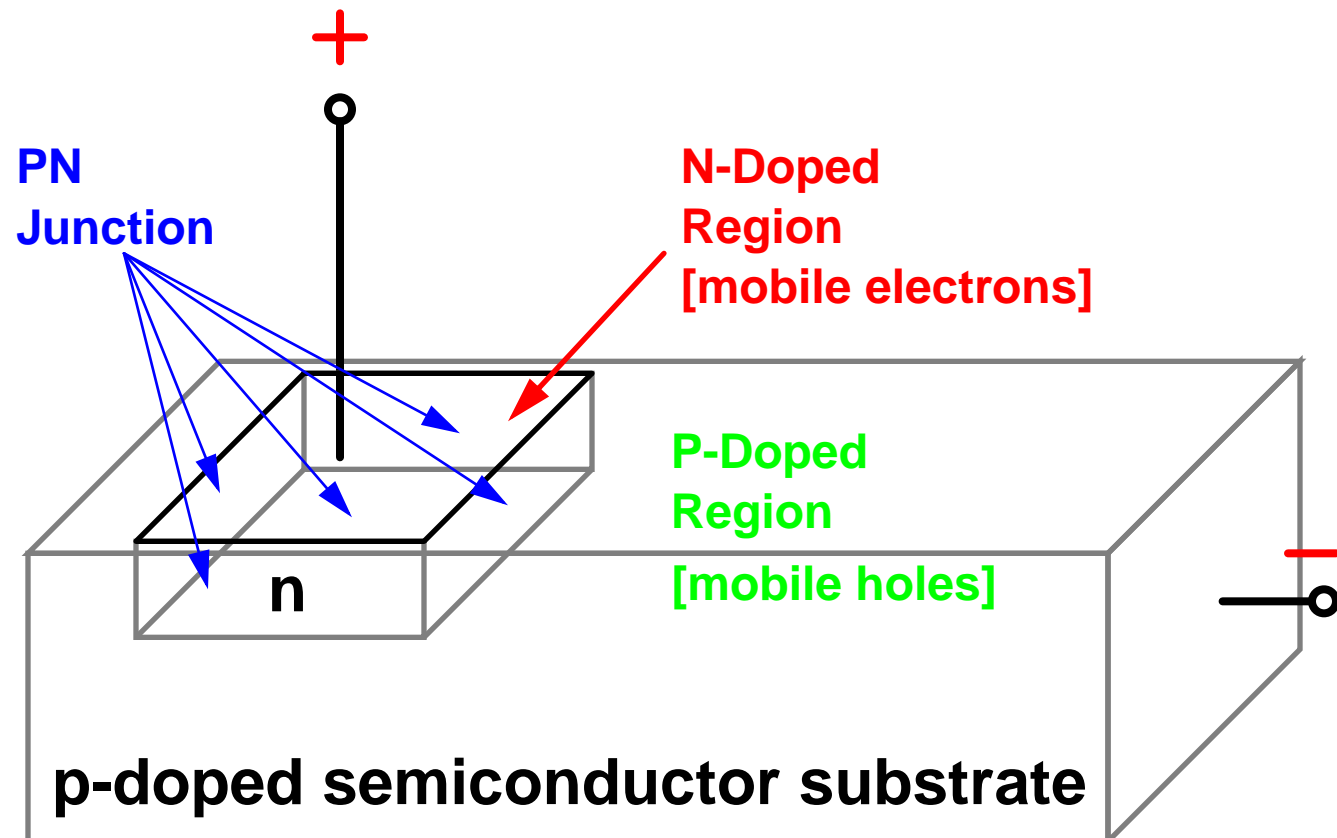
The P/N Junction

BIAS EFFECT on DEPLETION REGION



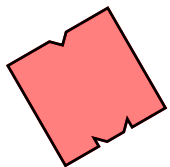
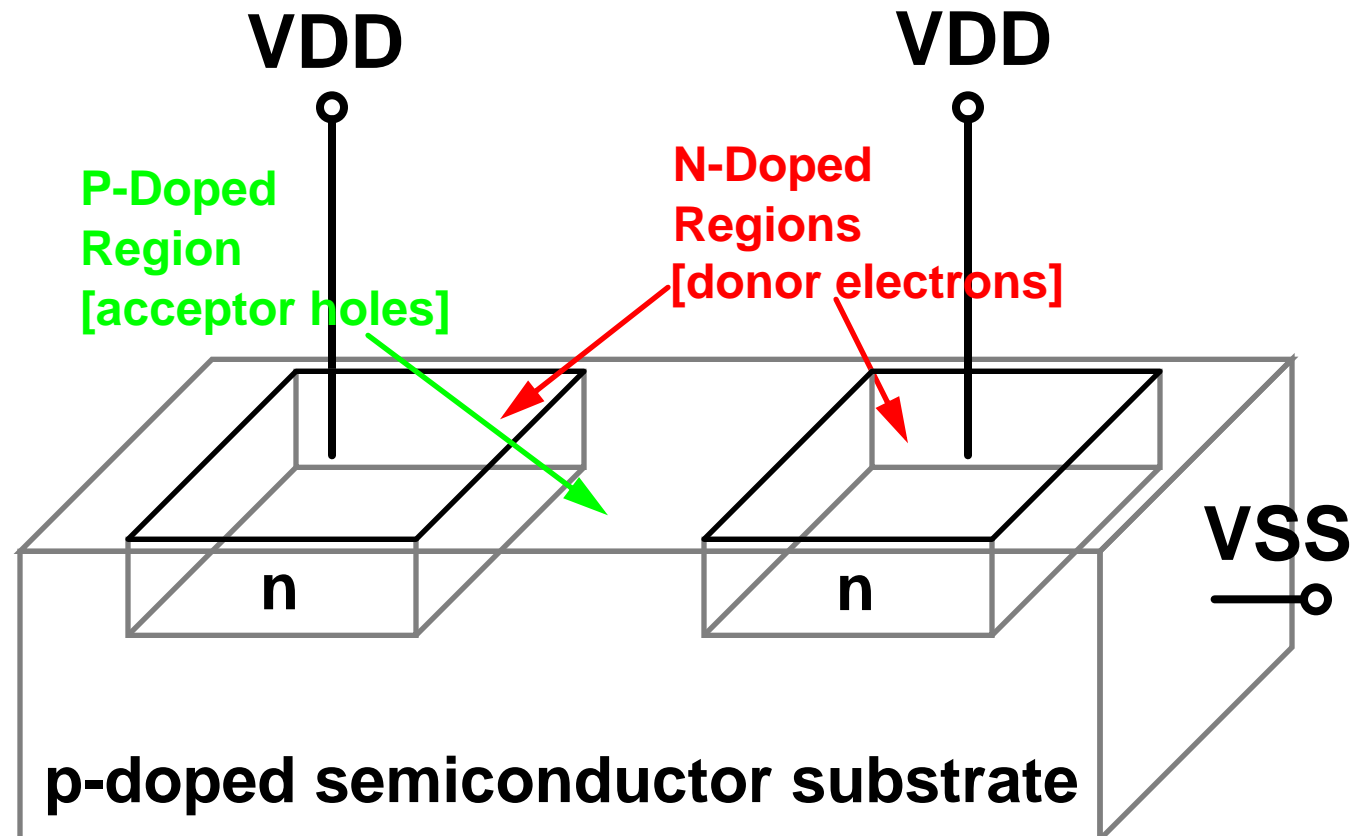
MOS Transistors

MOS Transistor, reverse-biased:



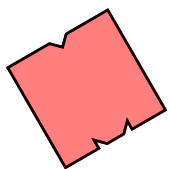
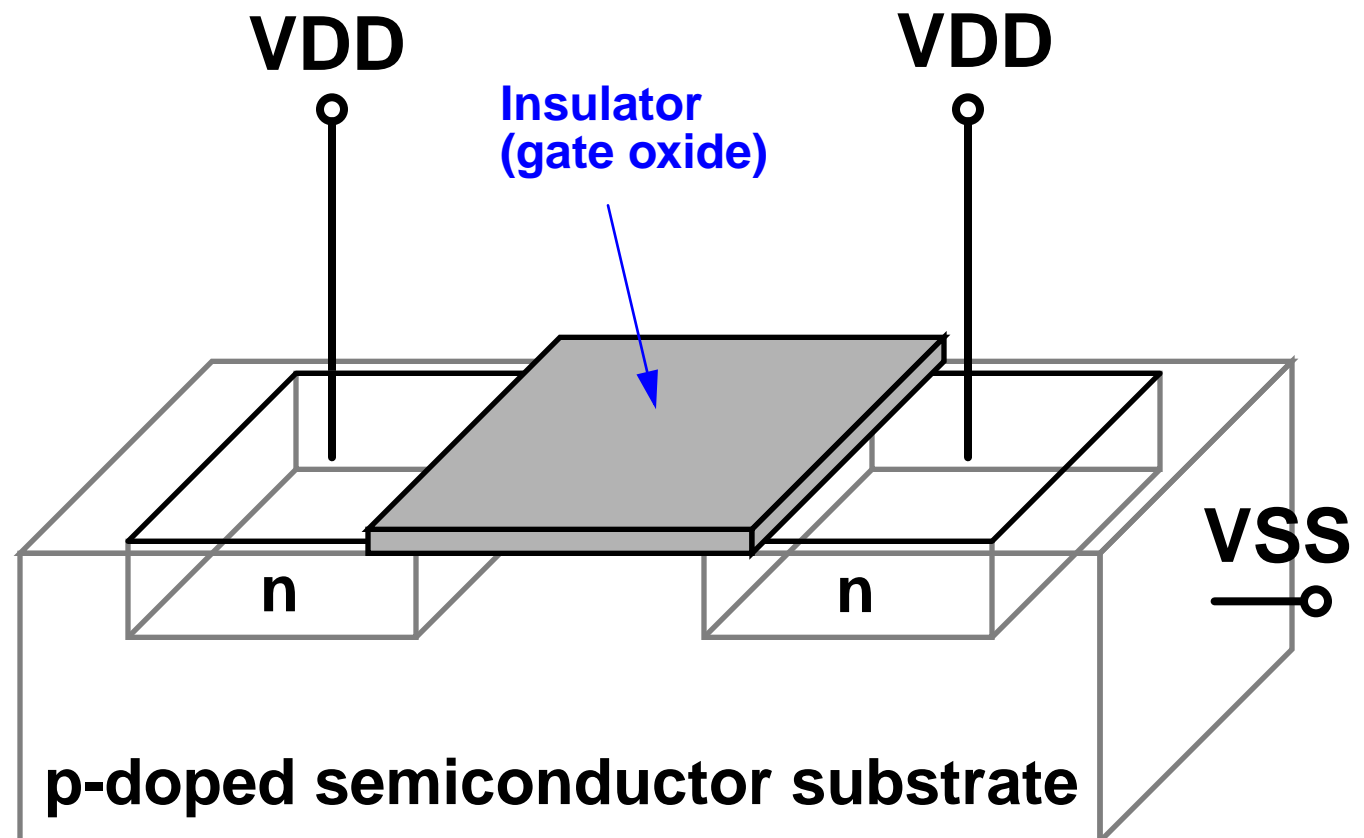
MOS Transistors

MOS Transistor, reverse-biased:



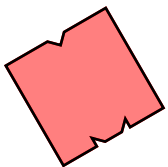
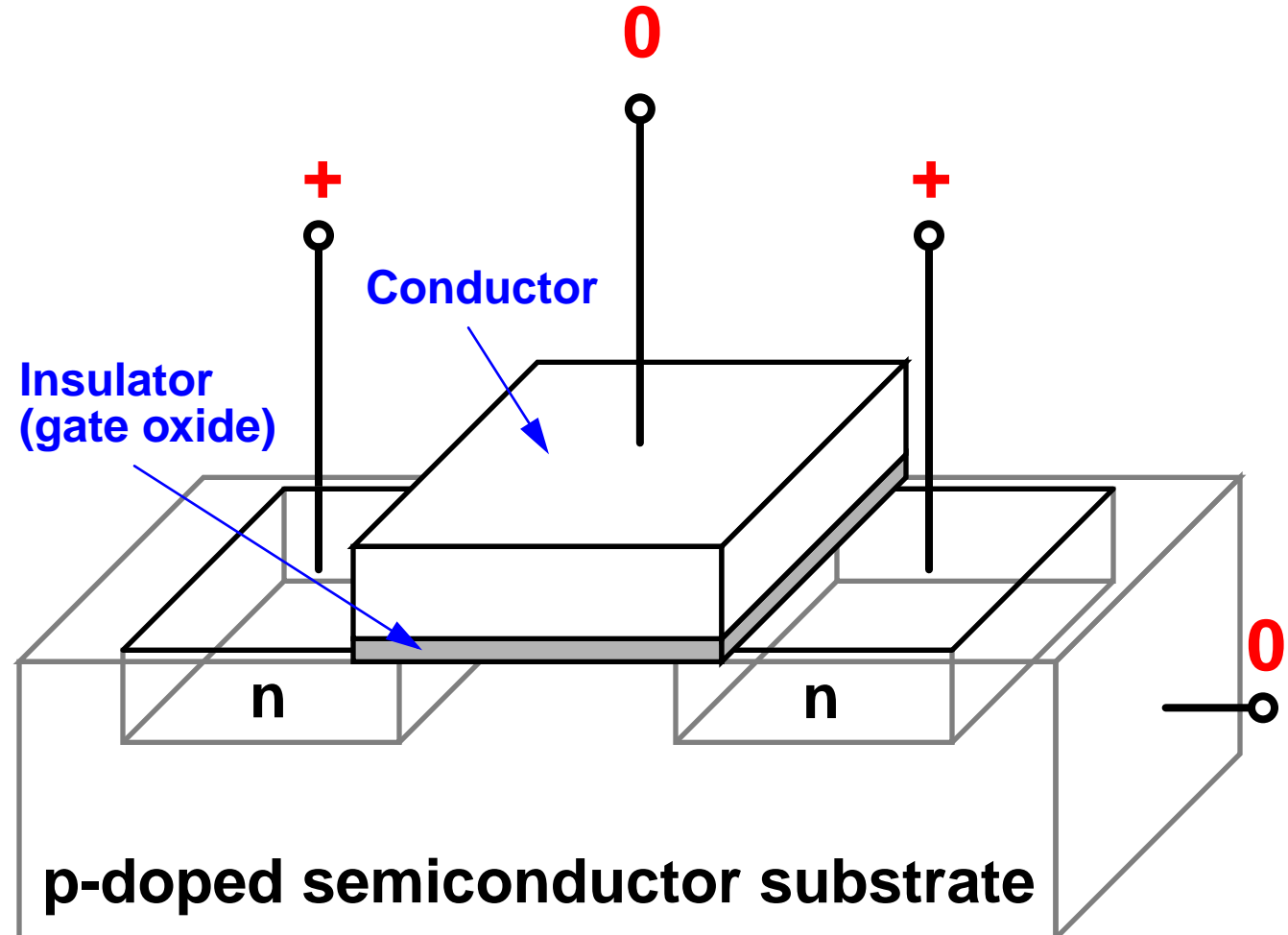
MOS Transistors

MOS Transistor, reverse-biased:



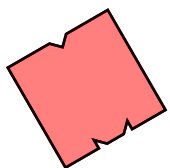
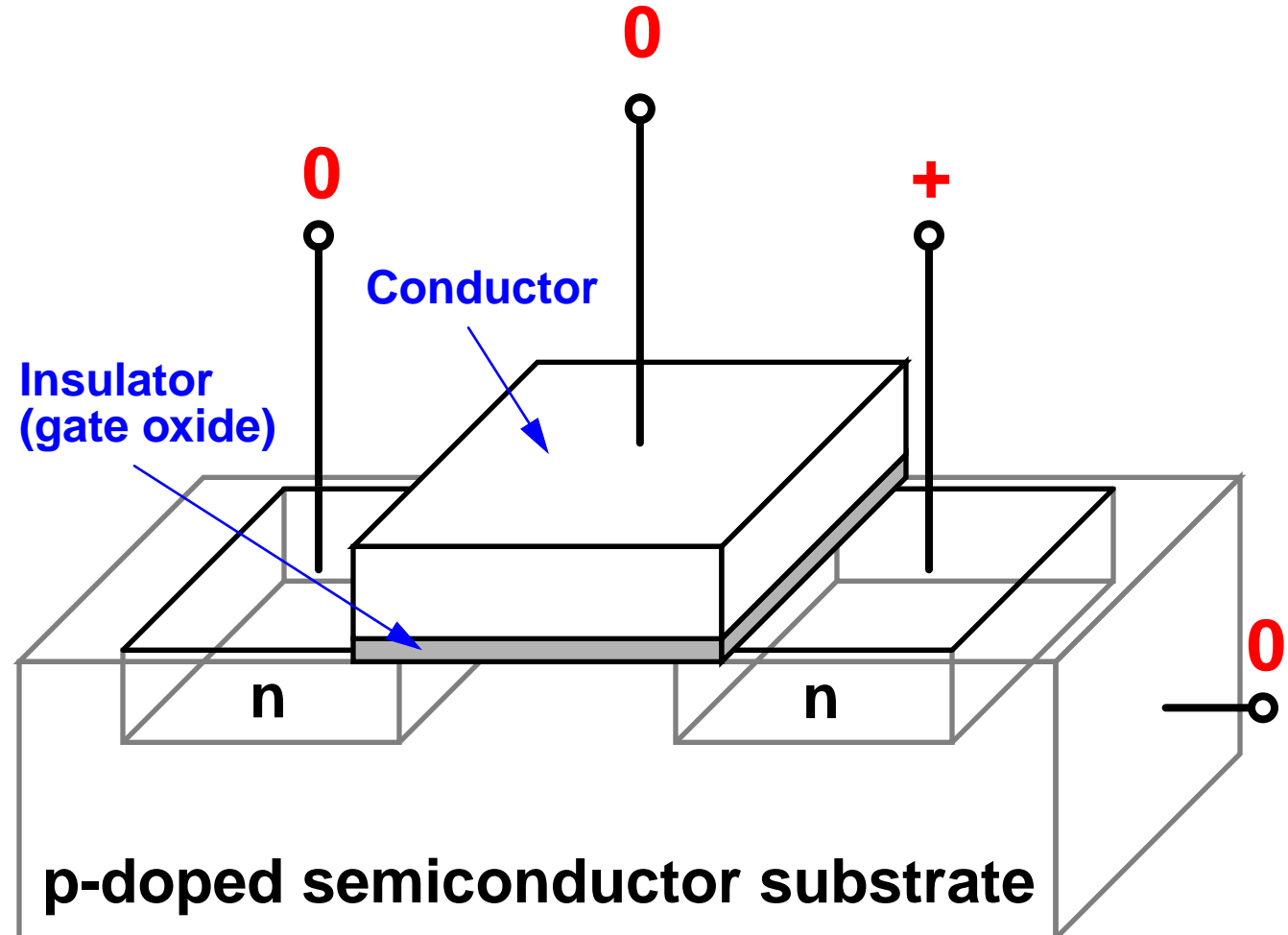
MOS Transistors

NMOS Transistor with gate:



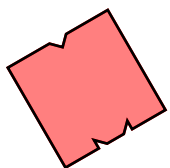
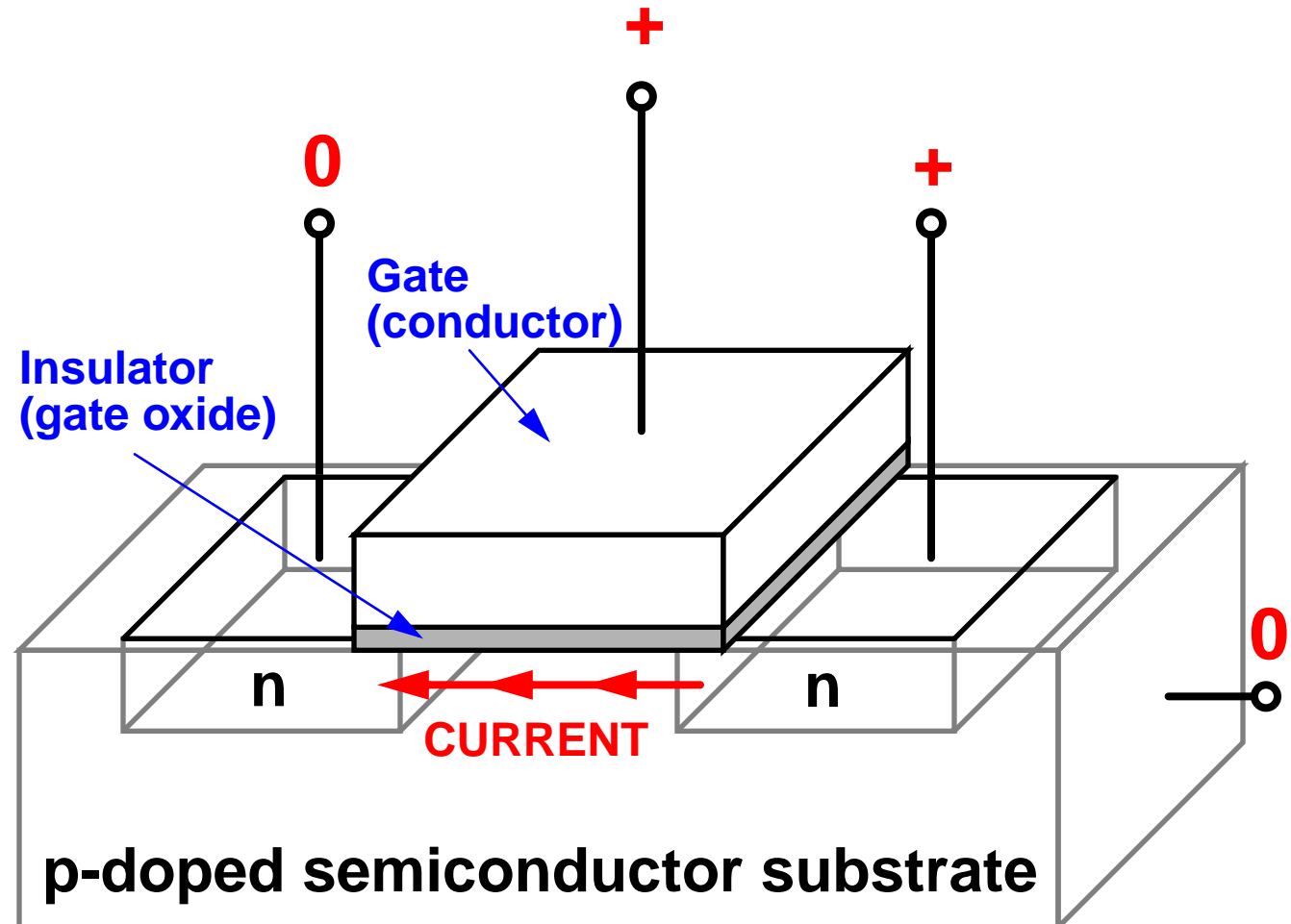
MOS Transistors

NMOS Transistor with bias voltages:



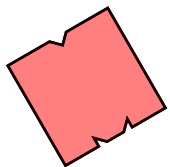
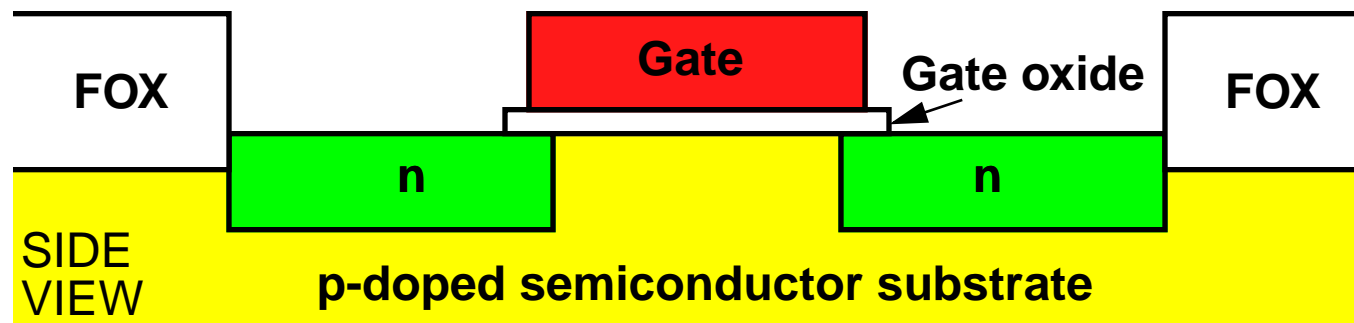
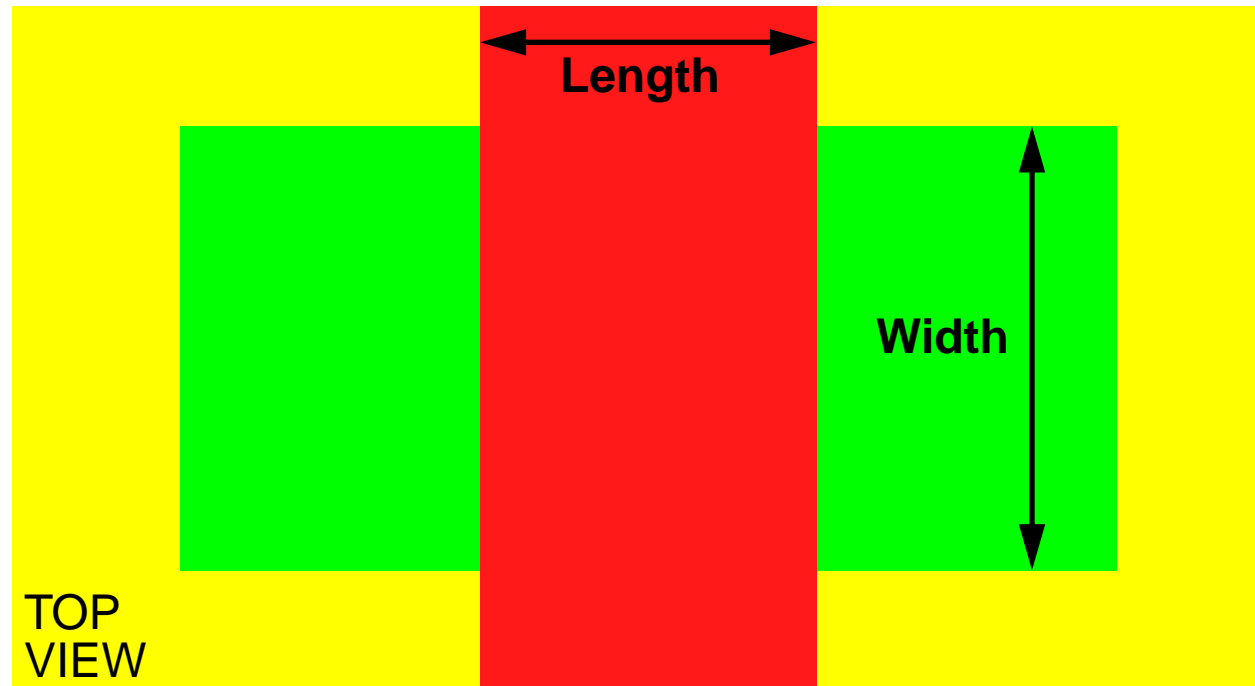
MOS Transistors

NMOS Transistor with bias voltages:



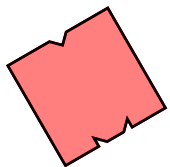
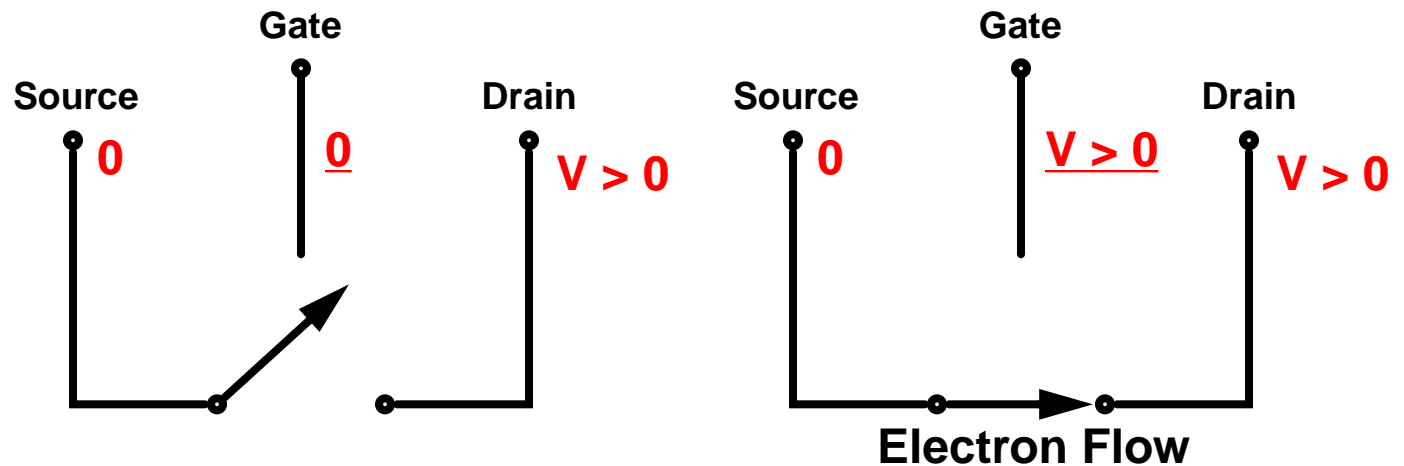
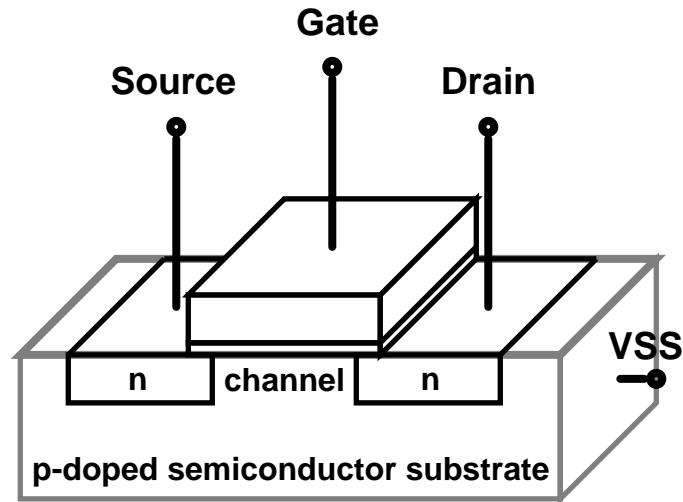
MOS Transistors

NMOS Transistor, two views:



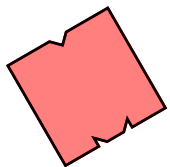
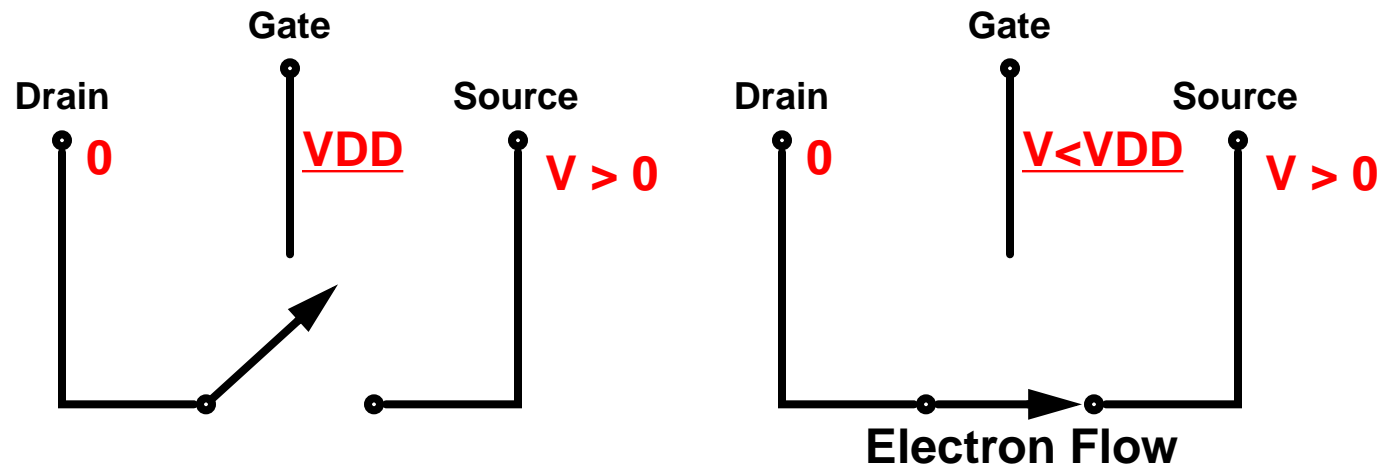
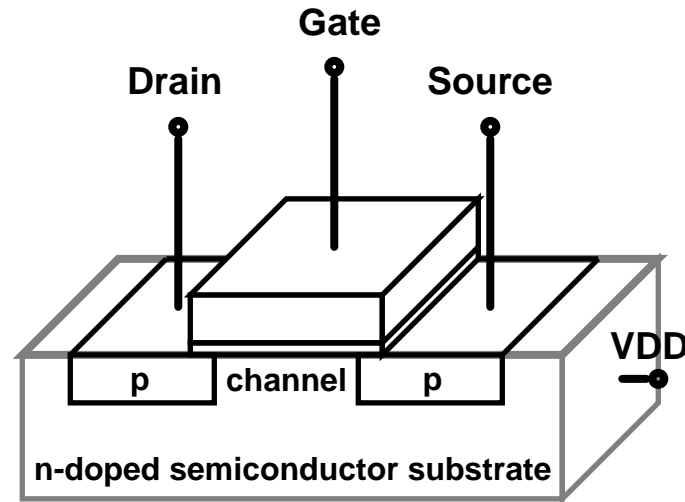
MOS Transistors

NMOS Transistor with bias voltages:



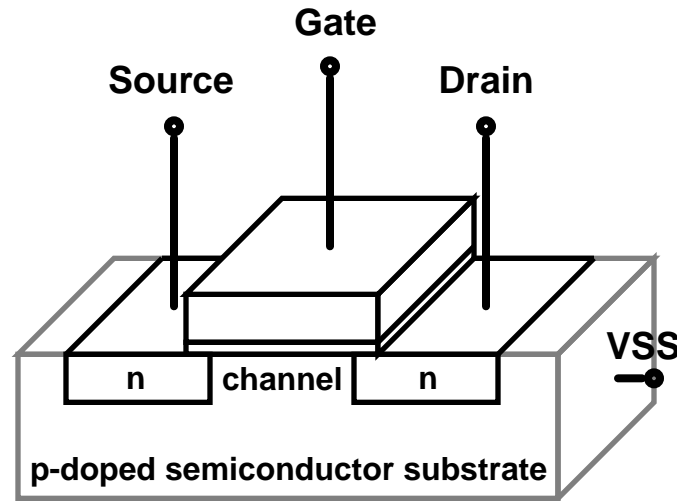
MOS Transistors

PMOS Transistor with bias voltages:

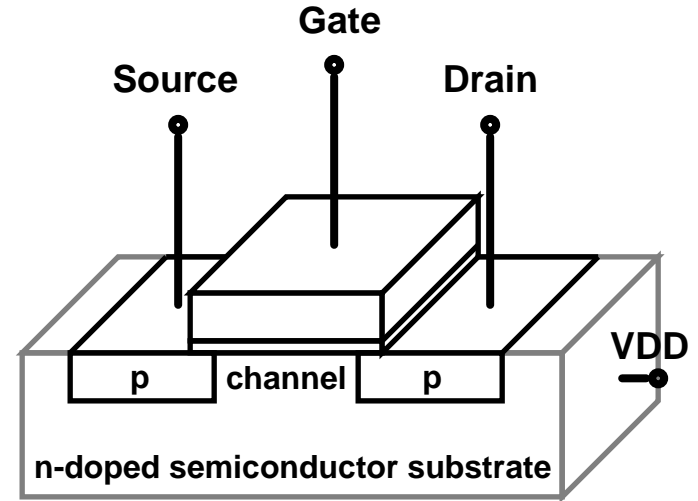
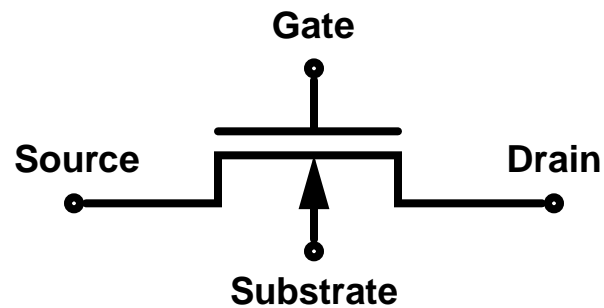


MOS Transistors

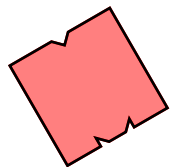
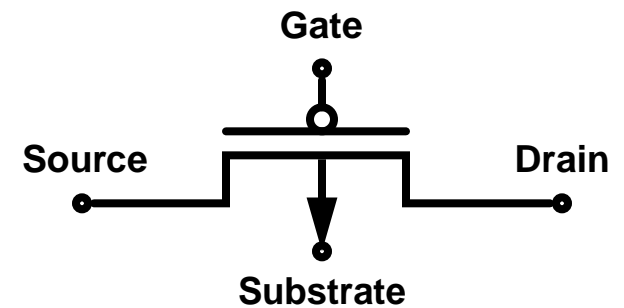
MOS Transistors:



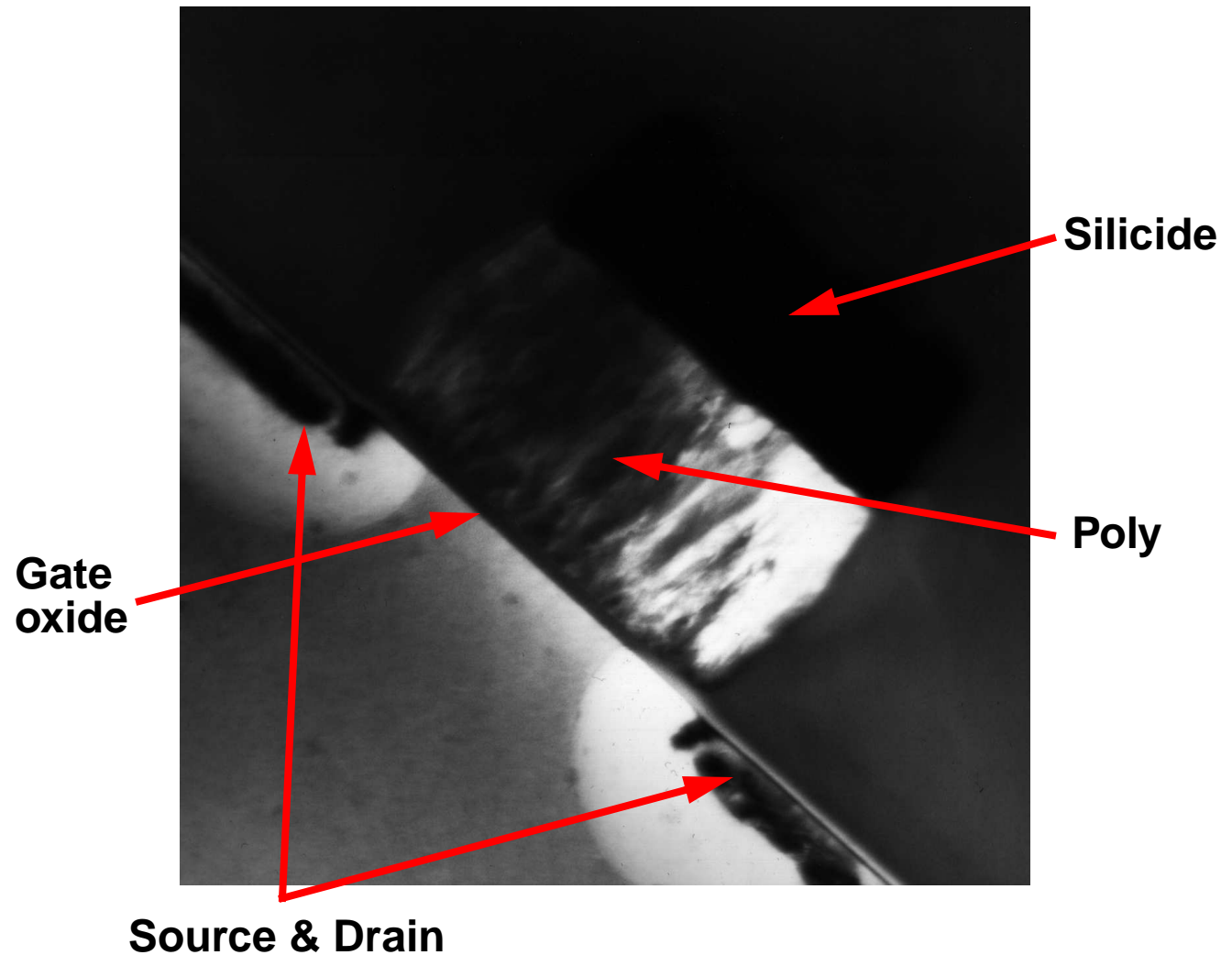
NMOS



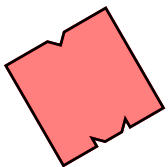
PMOS



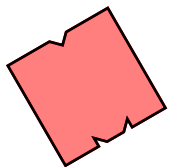
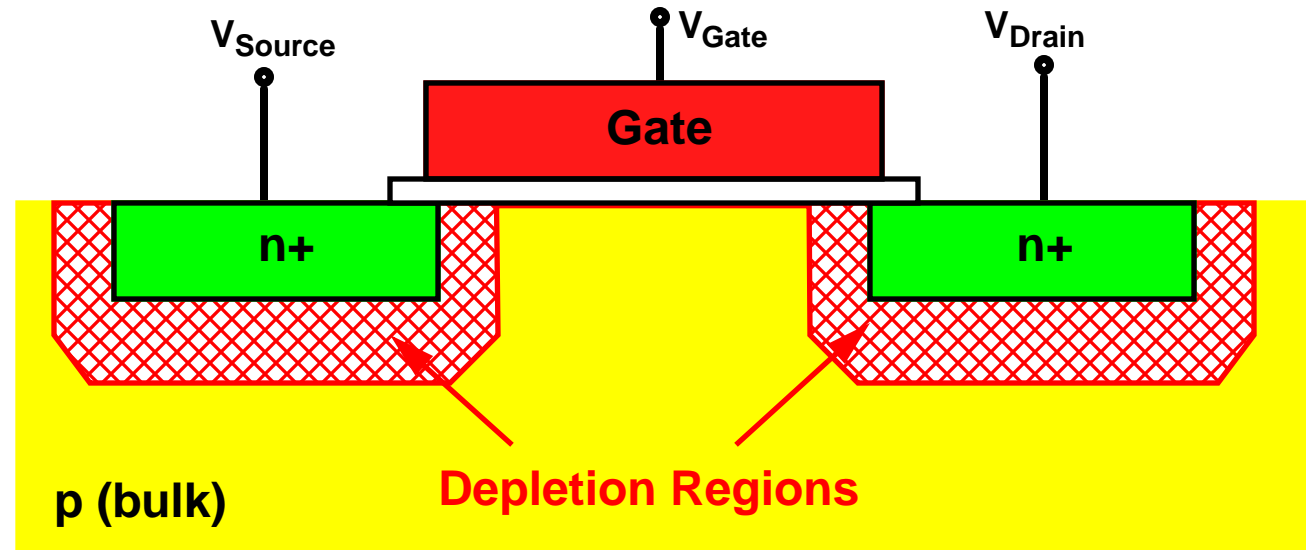
0.25 μm transistor (Bell Labs)



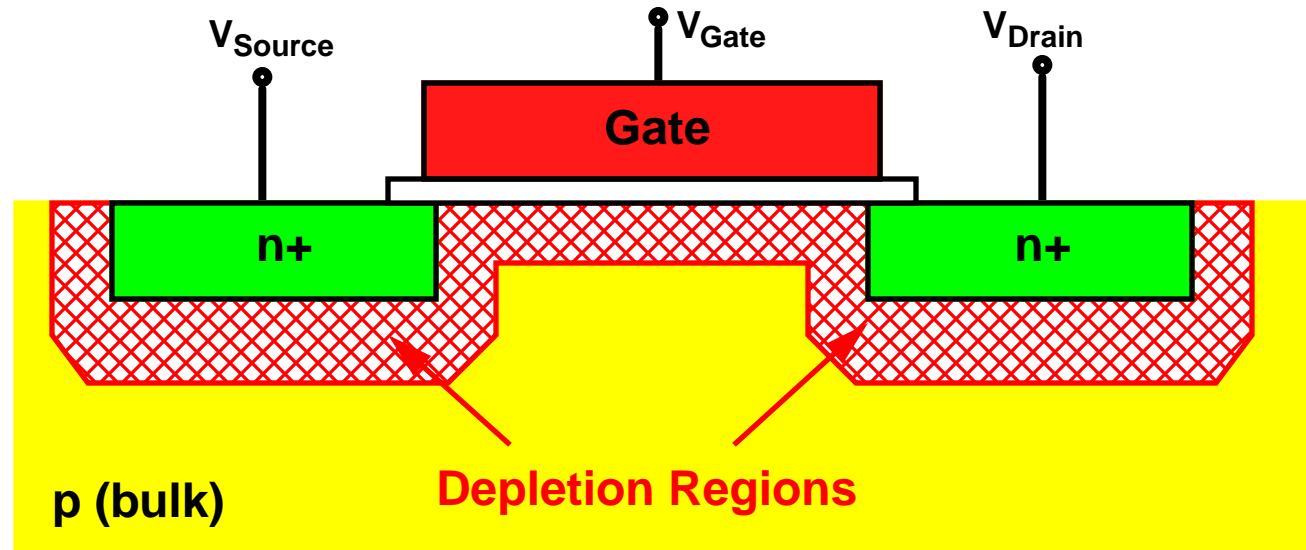
Poly+silicide = “polycide gate” (lower R)



MOS Behavior



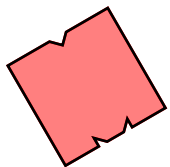
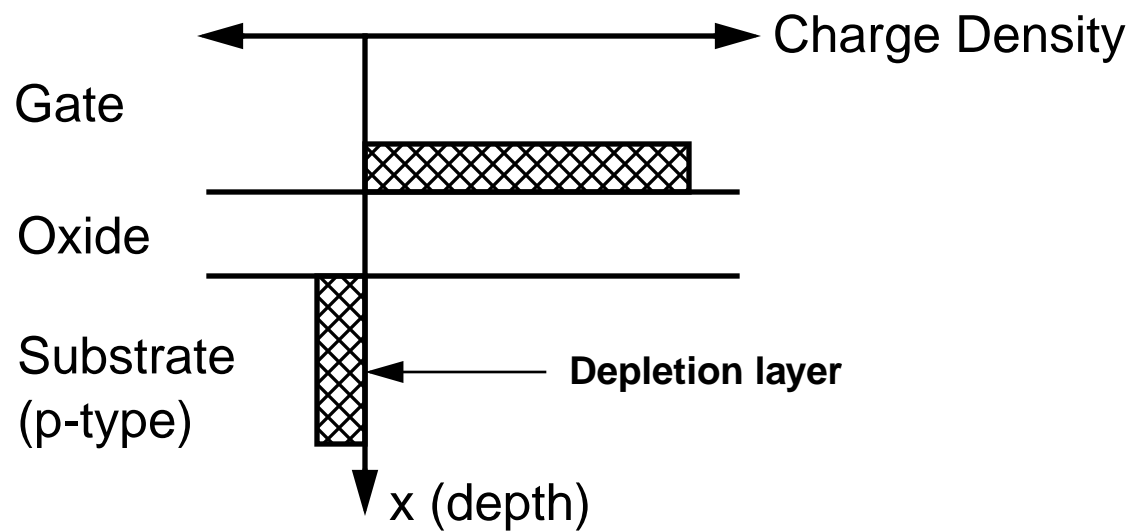
MOS Behavior



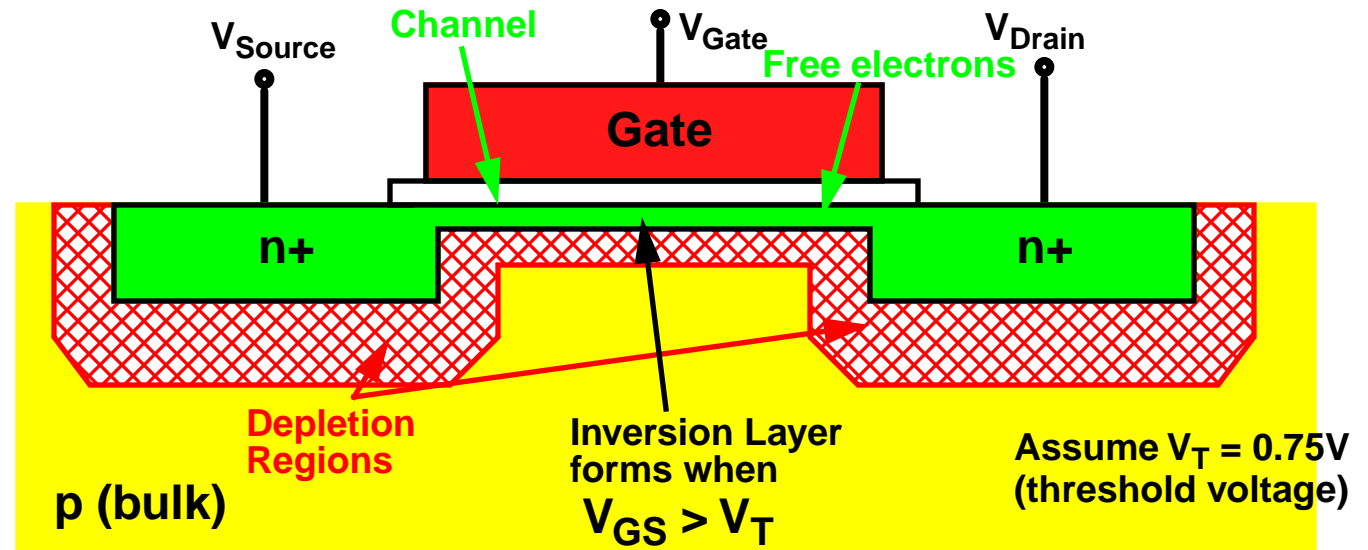
$$V_S = 0V$$

$$V_G = 0.5V$$

$$V_D = 0V$$



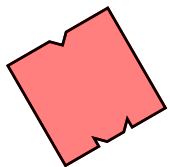
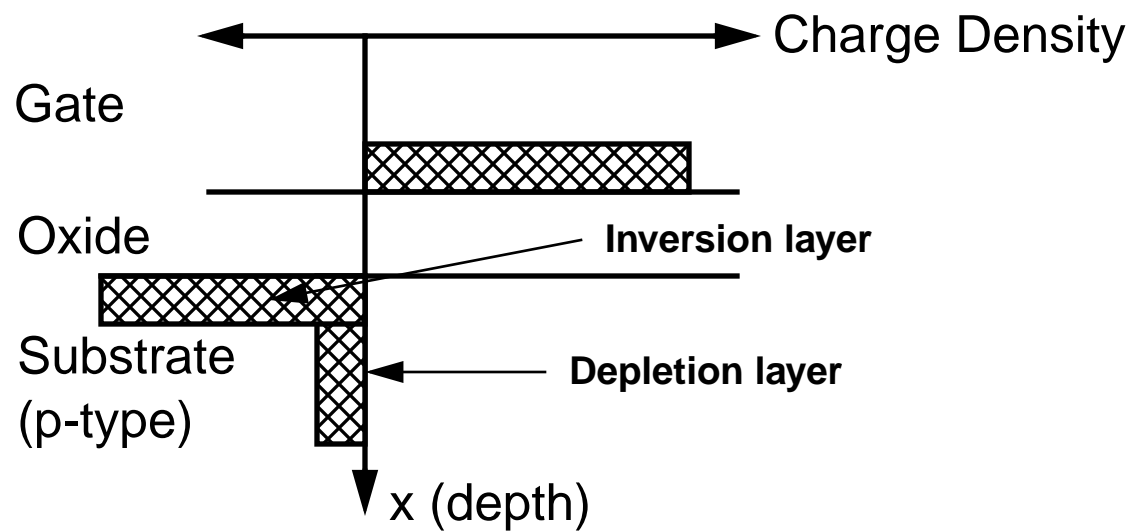
MOS Behavior



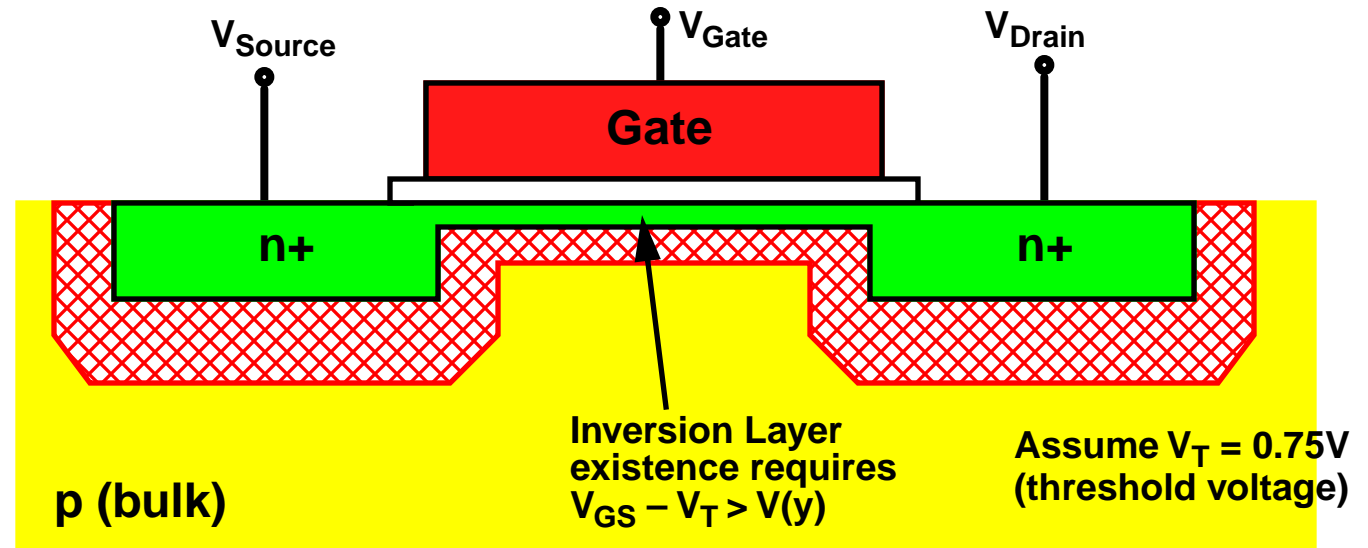
$$V_S = 0V$$

$$V_G = 1V$$

$$V_D = 0V$$



MOS Behavior: linear region



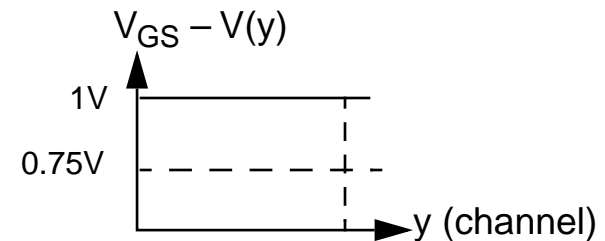
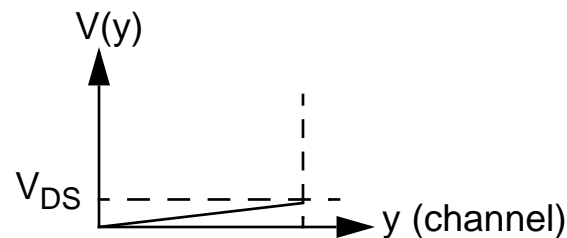
$$V_S = 0V$$

$$V_G = 1V$$

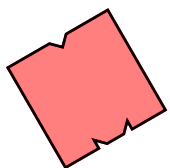
$$V_D = 0.001V$$

$$I_{DS} = \mu_n \frac{\epsilon_{OX}}{t_{OX}} \left(\frac{W}{L} \right) (V_{GS} - V_T) V_{DS}$$

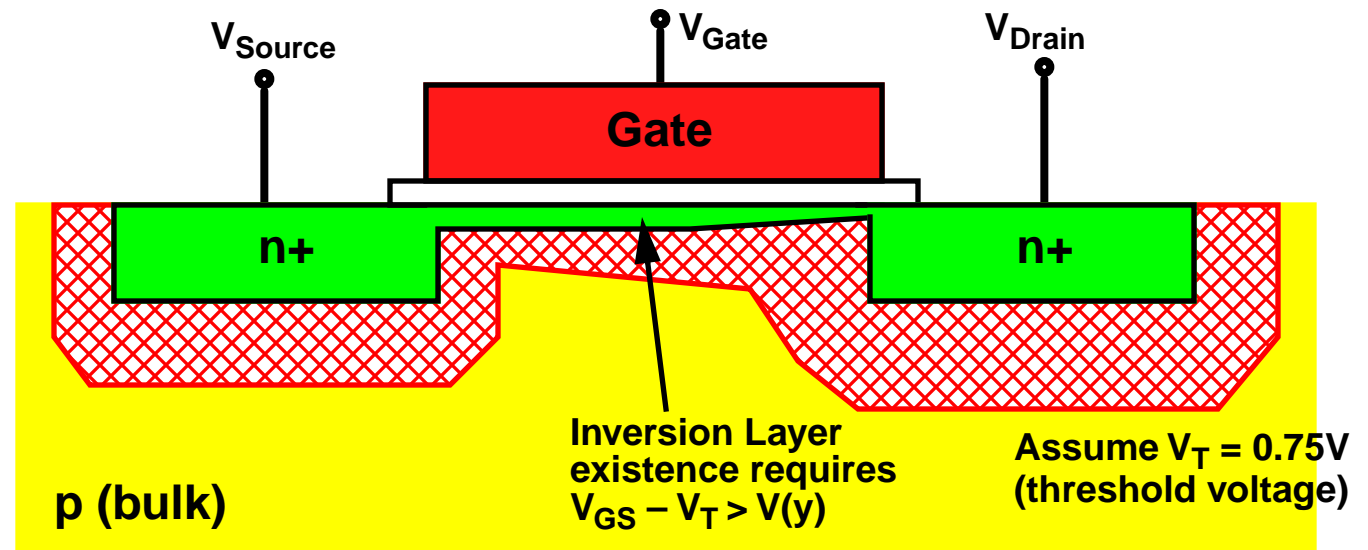
dielectric constant
electron mobility
oxide thickness



True when $V_{GS} > V_T$ & $V_{DS} \ll V_{GS} - V_T$



MOS Behavior: 'linear' region

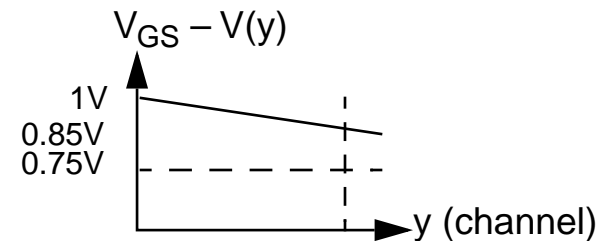
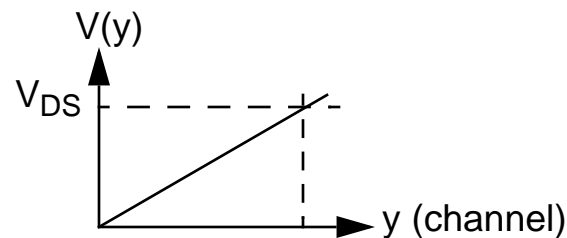


$$V_S = 0V$$

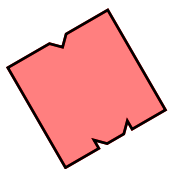
$$V_G = 1V$$

$$V_D = 0.15V$$

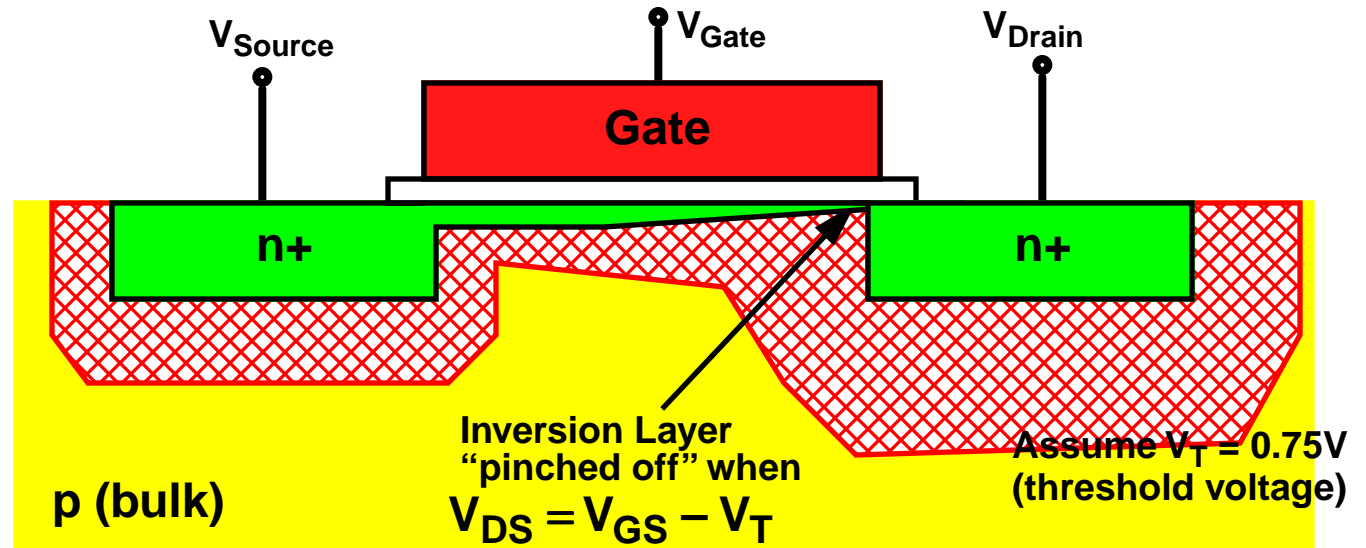
$$I_{DS} = \underbrace{\mu_n}_{\text{electron mobility}} \underbrace{\frac{\epsilon_{OX}}{t_{OX}}}_{\text{dielectric constant / oxide thickness}} \left(\frac{W}{L} \right) \left[(V_{GS} - V_T) V_{DS} - \frac{1}{2} V_{DS}^2 \right]$$



True when $V_{GS} > V_T$ & $V_{DS} \leq V_{GS} - V_T$



MOS Behavior: saturation

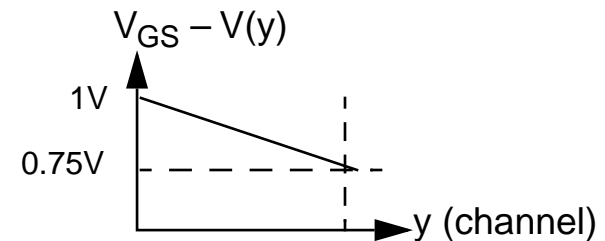
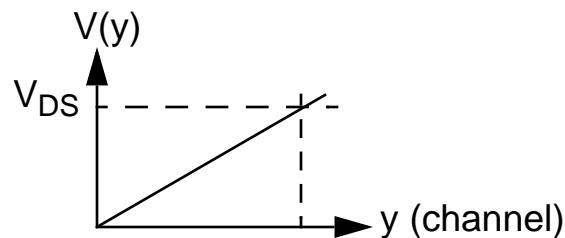


$$V_S = 0V$$

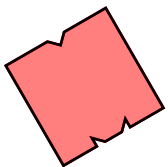
$$V_G = 1V$$

$$V_D = 0.25V$$

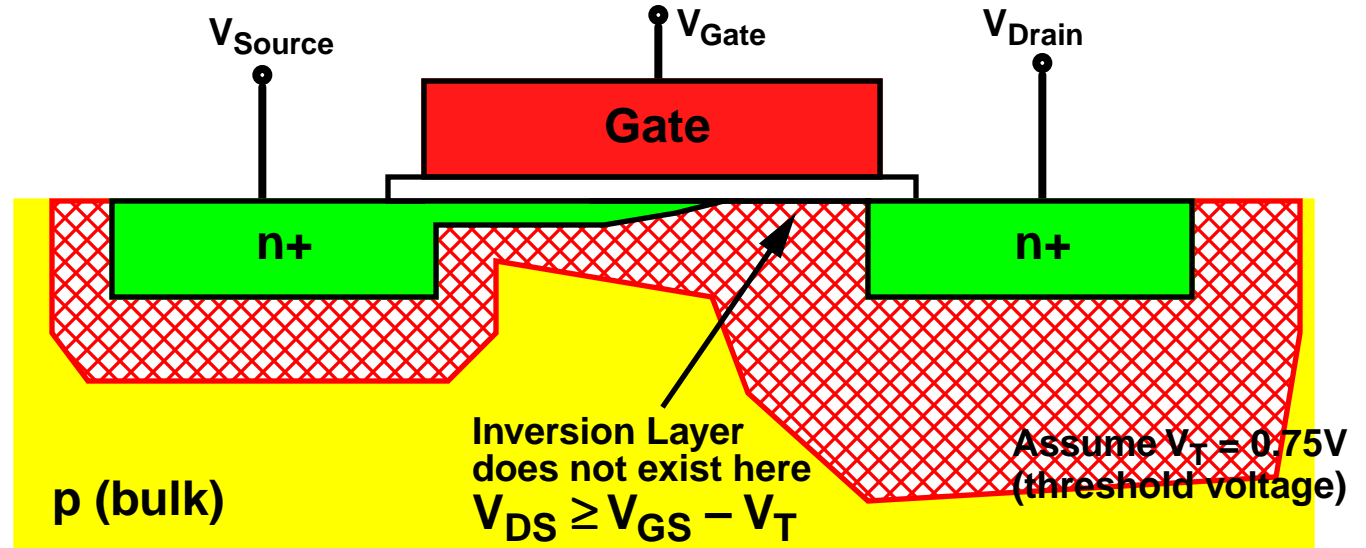
$$I_{DS} = \frac{1}{2} \left(\mu_n \frac{\epsilon_{OX}}{t_{OX}} \left(\frac{W}{L} \right) \right) (V_{GS} - V_T)^2$$



True when $V_{GS} > V_T$ & $V_{DS} = V_{GS} - V_T$



MOS Behavior: modulation

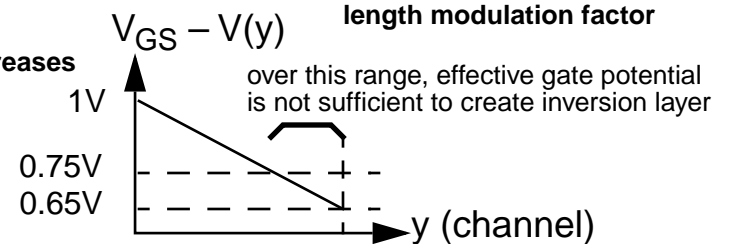
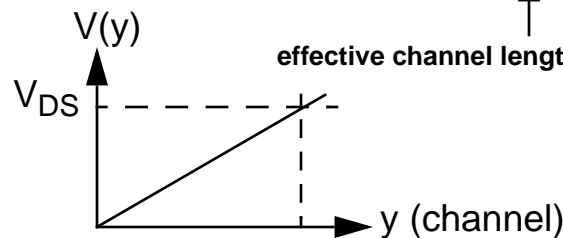


$$V_S = 0V$$

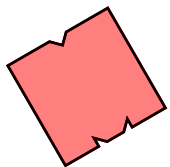
$$V_G = 1V$$

$$V_D = 0.35V$$

$$I_{DS} = \frac{1}{2} \left(\mu_n \frac{\epsilon_{OX}}{t_{OX}} \left(\frac{W}{L} \right) \right) (V_{GS} - V_T)^2 (1 + \lambda V_{DS})$$



True when $V_{GS} > V_T$ & $V_{DS} \geq V_{GS} - V_T$



Example of Drain Current

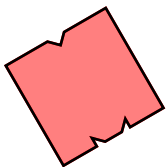
Values for generic 0.5 μm process:

	k' (transconductance) = $\mu_n \frac{\epsilon_{\text{ox}}}{t_{\text{ox}}}$	V_T
n-type	$k'_n = 73 \mu\text{A}/\text{V}^2$	0.7V
p-type	$k'_p = 21 \mu\text{A}/\text{V}^2$	-0.8V

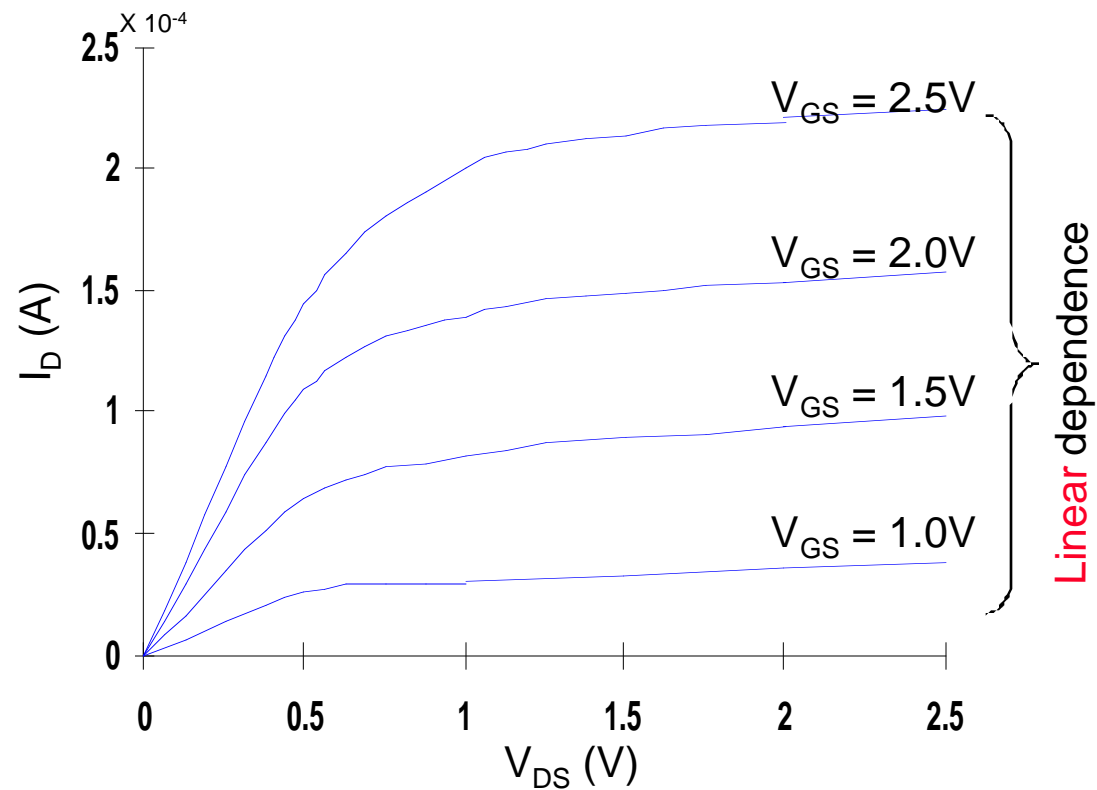
Assume $W/L = 3/2$, $V_{GS} = 2\text{V}$, find I_{DS} for NMOS device at saturation point:

$$I_{DS} = \frac{1}{2} \left(k' \frac{W}{L} \right) (V_{GS} - V_T)^2$$

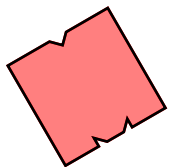
$$I_{DS} = \frac{1}{2} \left(73 \frac{\mu\text{A}}{\text{V}^2} \right) \left(\frac{3}{2} \right) (2\text{V} - 0.7\text{V})^2 = 93 \mu\text{A}$$



NMOS I-V Plot

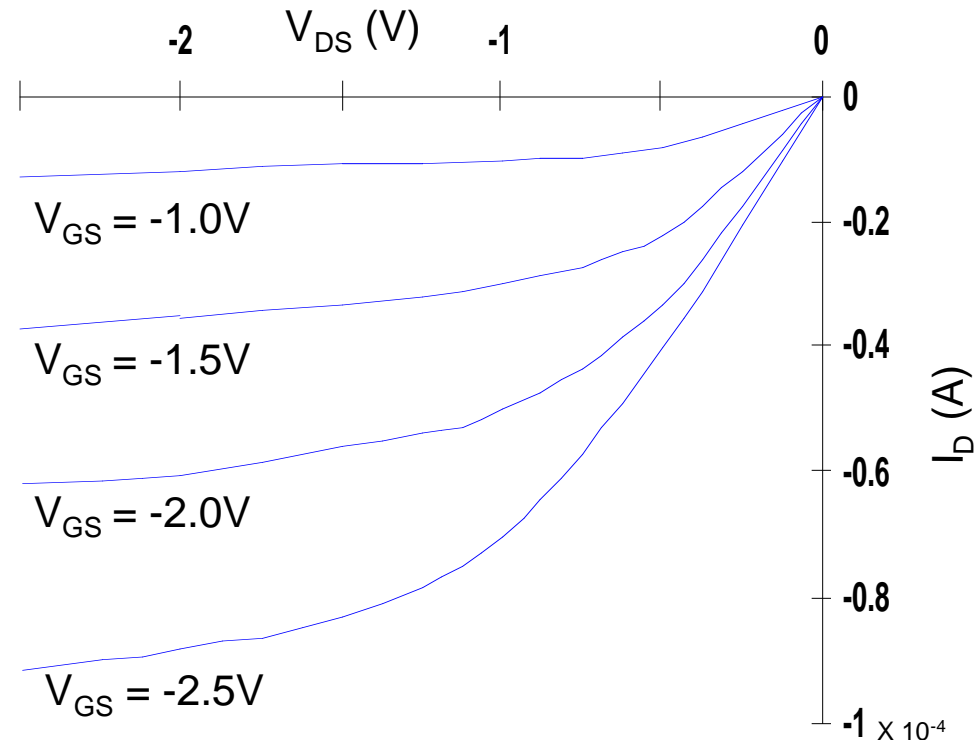


NMOS transistor, 0.25 μ m, $L_d = 0.25\mu$ m, $W/L = 1.5$, $V_{DD} = 2.5V$, $V_T = 0.4V$



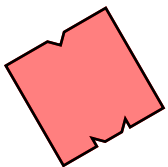
PMOS I-V Plot

| All polarities of all voltages and currents are reversed

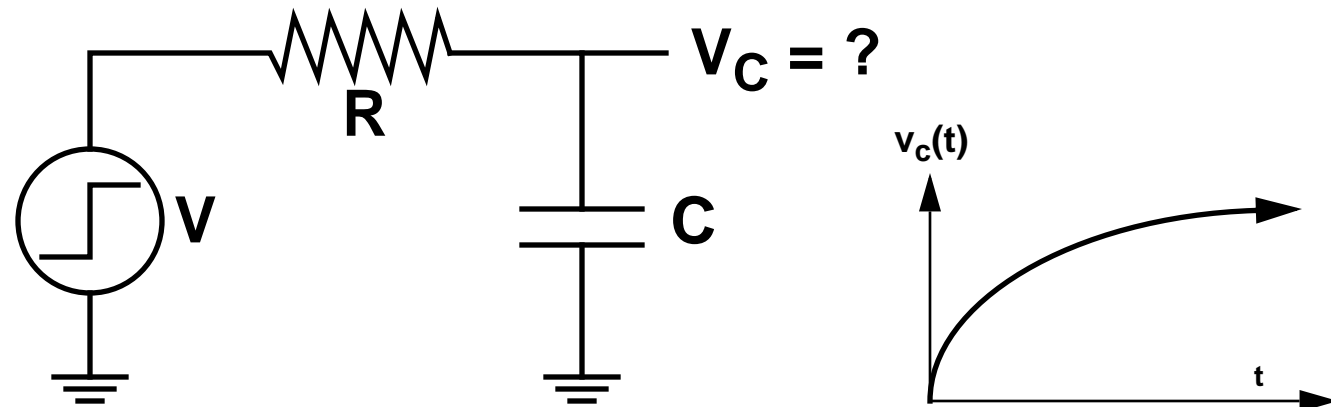


PMOS transistor, $0.25\mu\text{m}$, $L_d = 0.25\mu\text{m}$, $W/L = 1.5$, $V_{DD} = 2.5V$, $V_T = -0.4V$

Note y-axis scale (because $W/L_p = W/L_n$)
(drive current: I_D when $V_{GS} = V_{DS} = V_{DD}$)



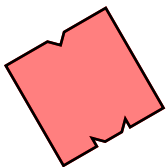
Review: RC Circuits



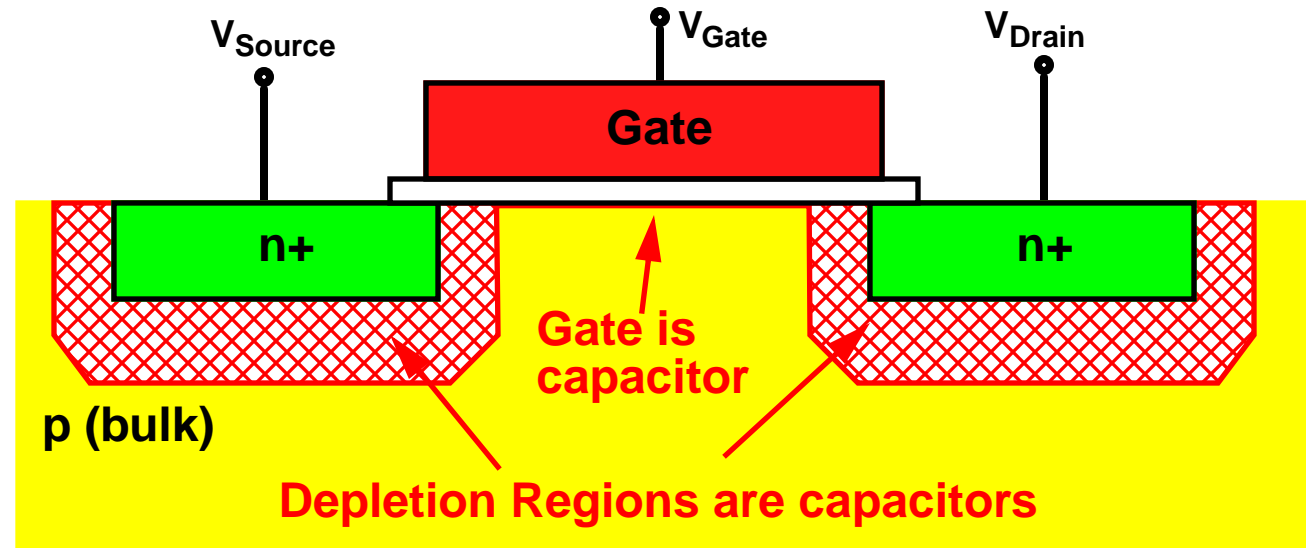
$$v_{\text{out}}(t) = (1 - e^{-t/\tau})V \quad \tau = RC$$

RC time-constant: dictates how rapidly the output voltage reacts to the voltage rise on input (step function).

Larger RC, slower response



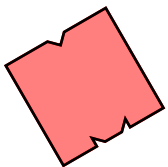
Capacitances



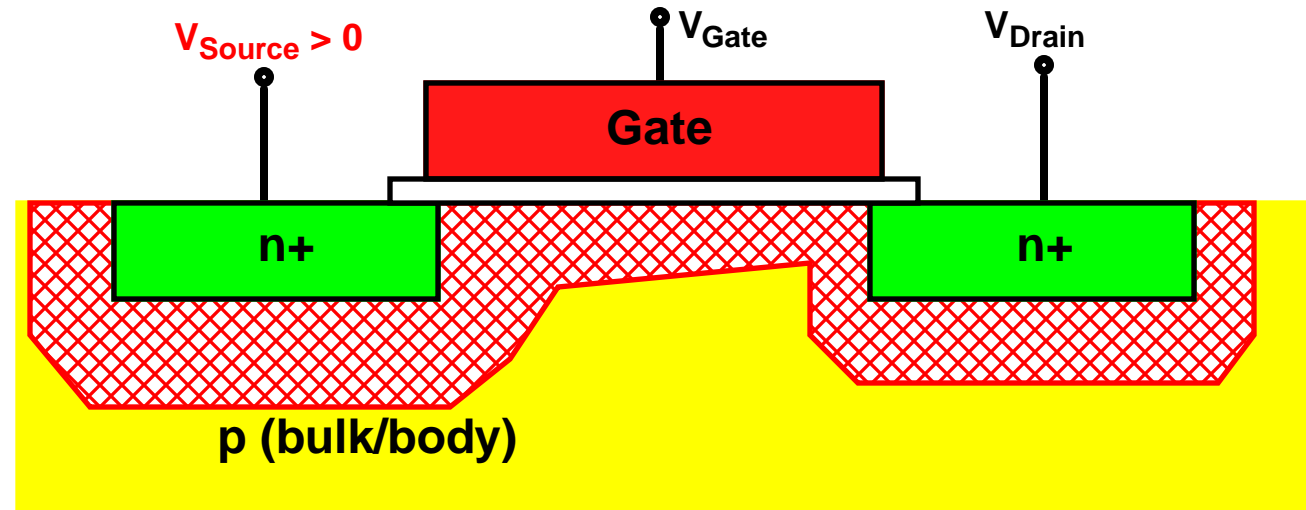
$$C_{gate} = \frac{WL\epsilon_{ox}}{t_{ox}} \quad C_{SC} = \frac{A\epsilon_{si}}{t_{si}} \quad C_{diff} = \frac{I \cdot \tau_c}{V_{th}}$$

Yes, there are others ...

Result: parasitic capacitances hinder
switching speeds



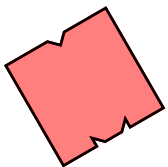
Body Effect



- Suppose **source** and **body** are not in equilibrium: reverse bias increases size of depletion region around that diode (and changes its parasitic capacitance)
- Called “body effect” ... it changes the threshold voltage for that device

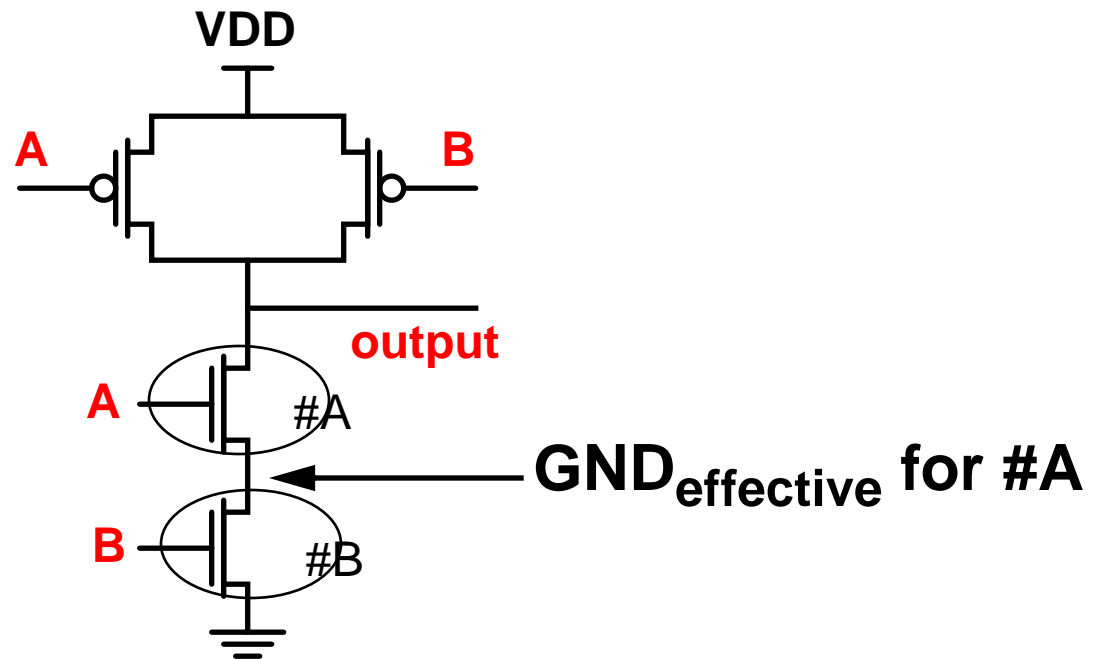
$$\Delta V_t = \frac{\sqrt{2q\epsilon_{si}N_A}}{C_{ox}} (\sqrt{\phi_S + V_{SB}} - \sqrt{\phi_S})$$

But can it happen?

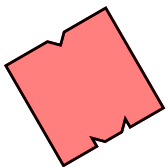


Body Effect

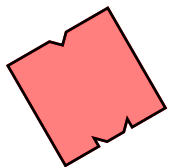
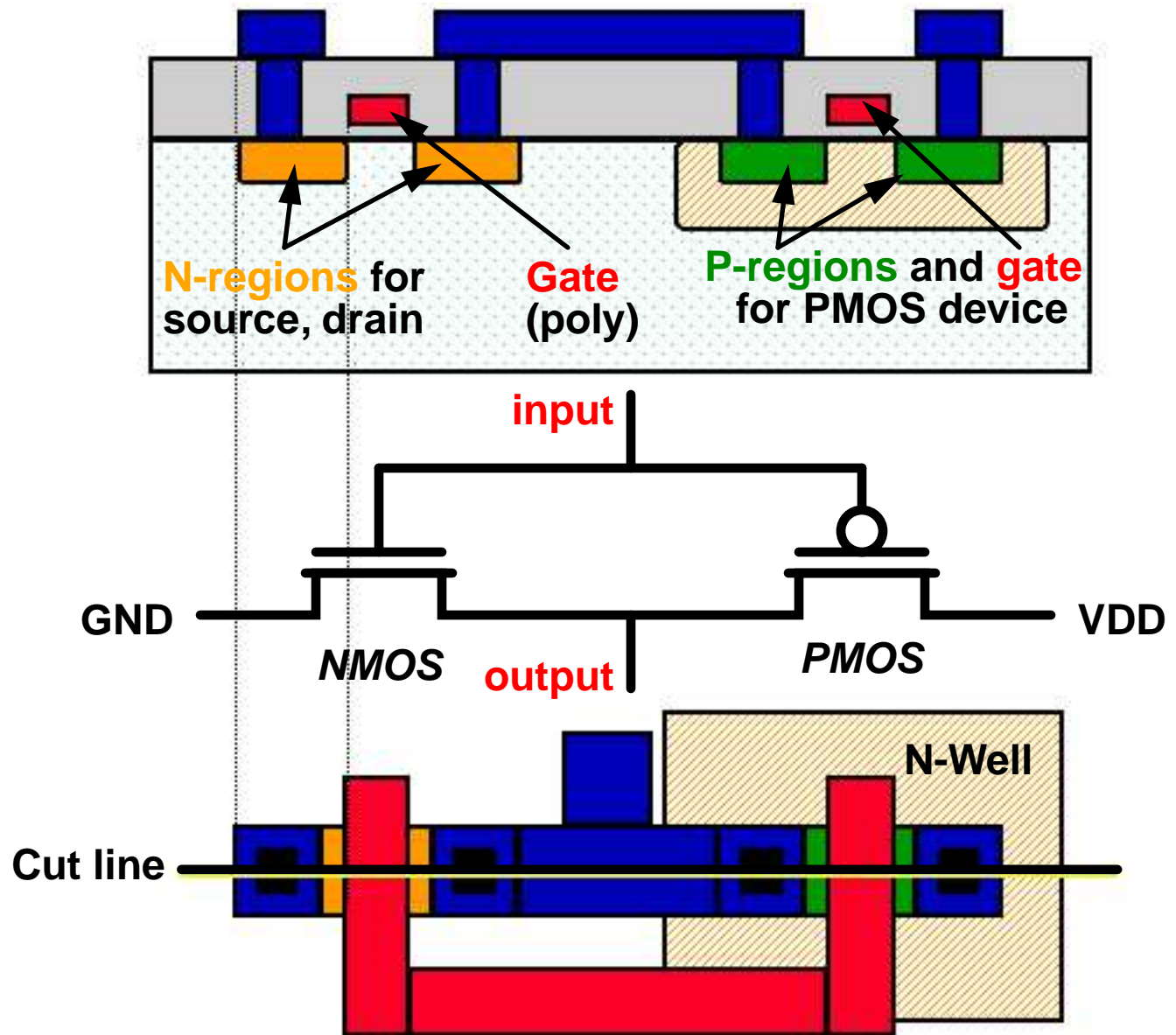
NAND gate



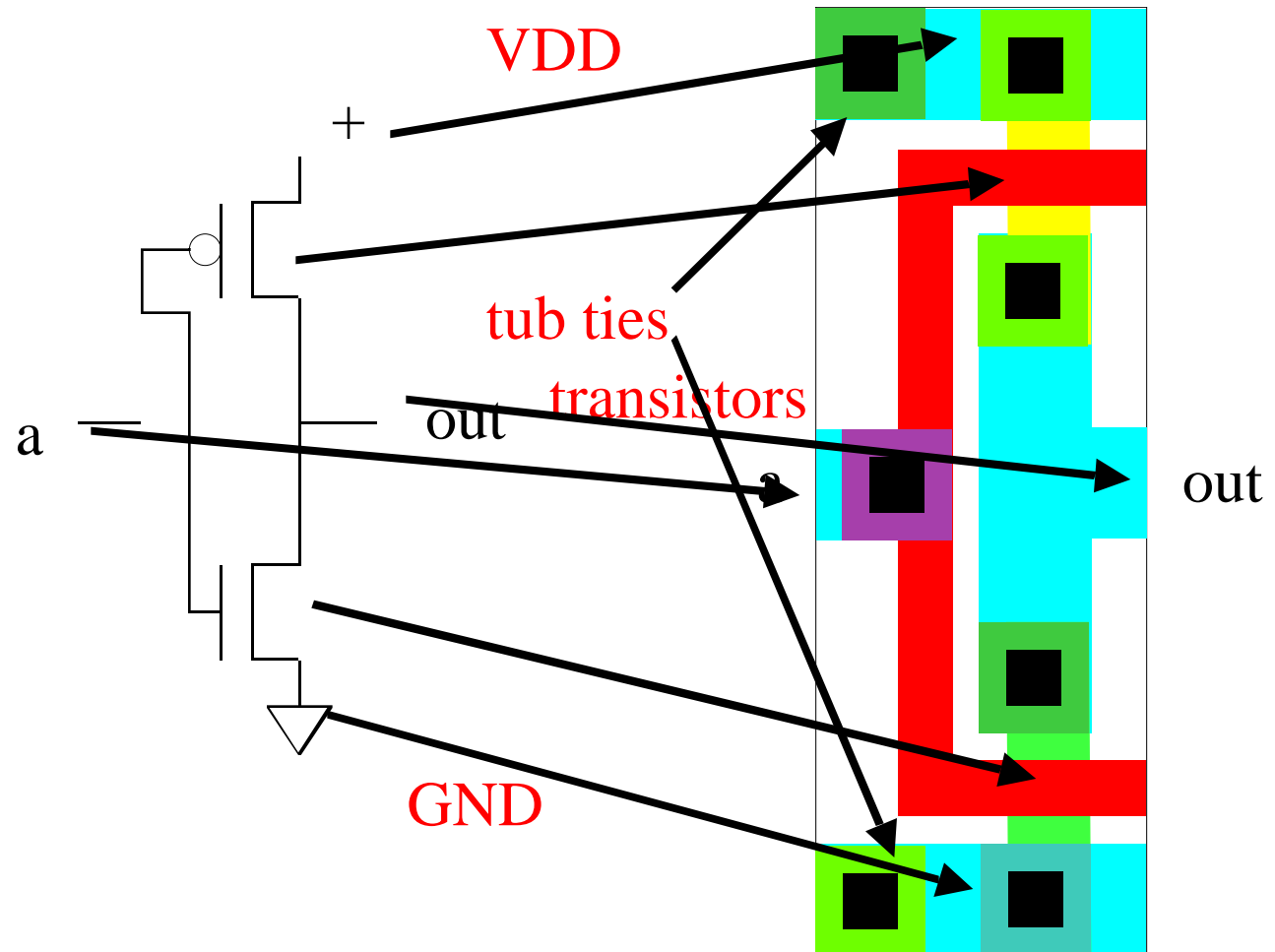
- If #B propagates signal in non-zero time, the effective source voltage for #A can go positive (higher than ground)
- Perspective: Things start to get interesting when you start connecting these things together ...



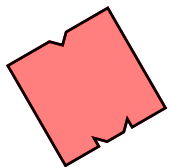
CMOS Inverter Layout I



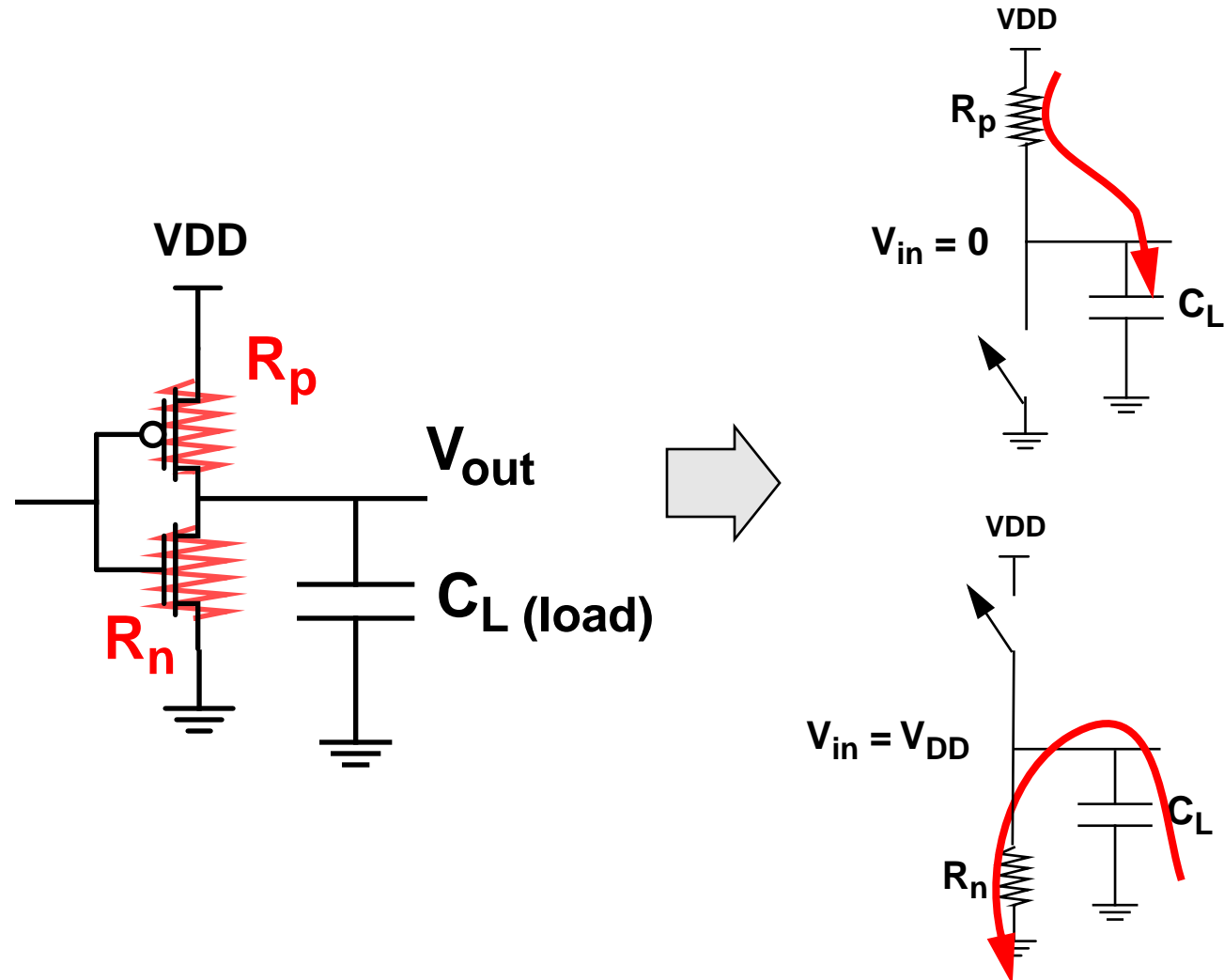
CMOS Inverter Layout II



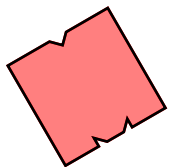
Another view (note: wells/tubs not shown)



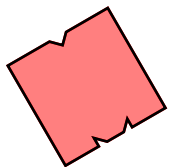
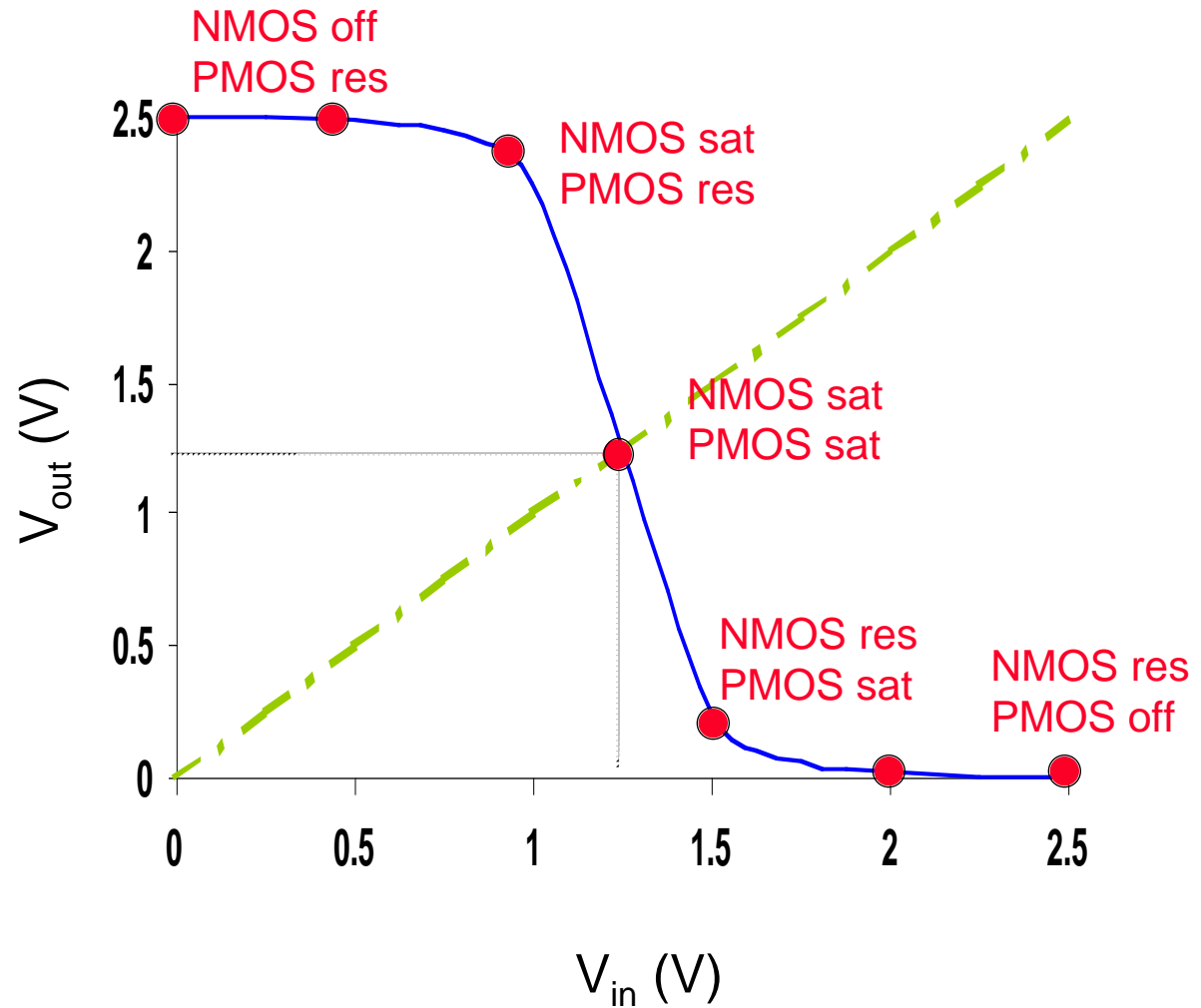
CMOS Inverter: Analysis



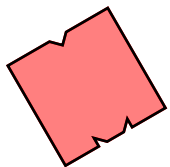
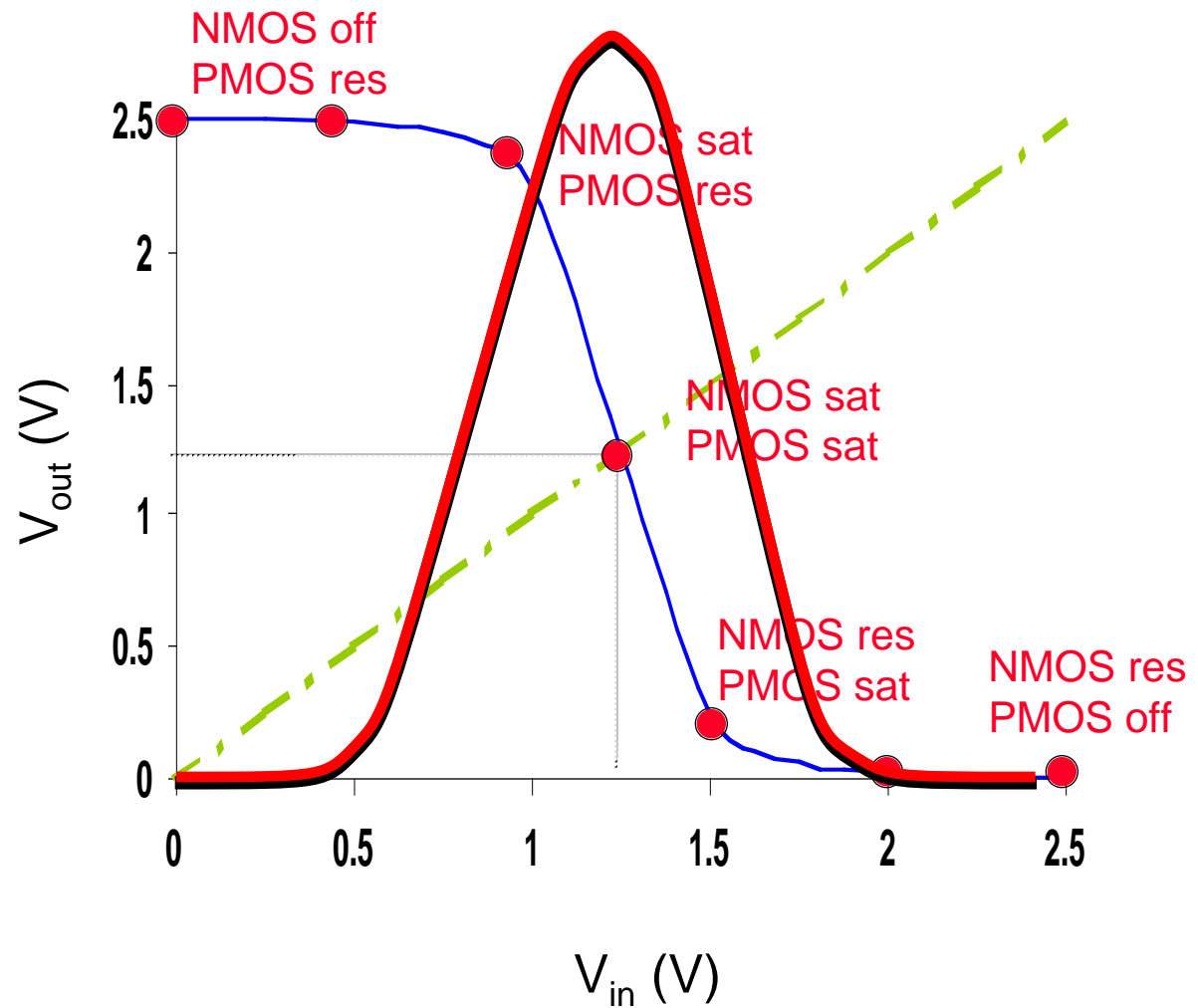
- Gate response time is determined by the time to charge C_L through R_p or discharge C_L through R_n



CMOS Inverter: Transfer Plot

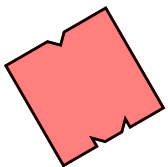


CMOS Inverter: Current

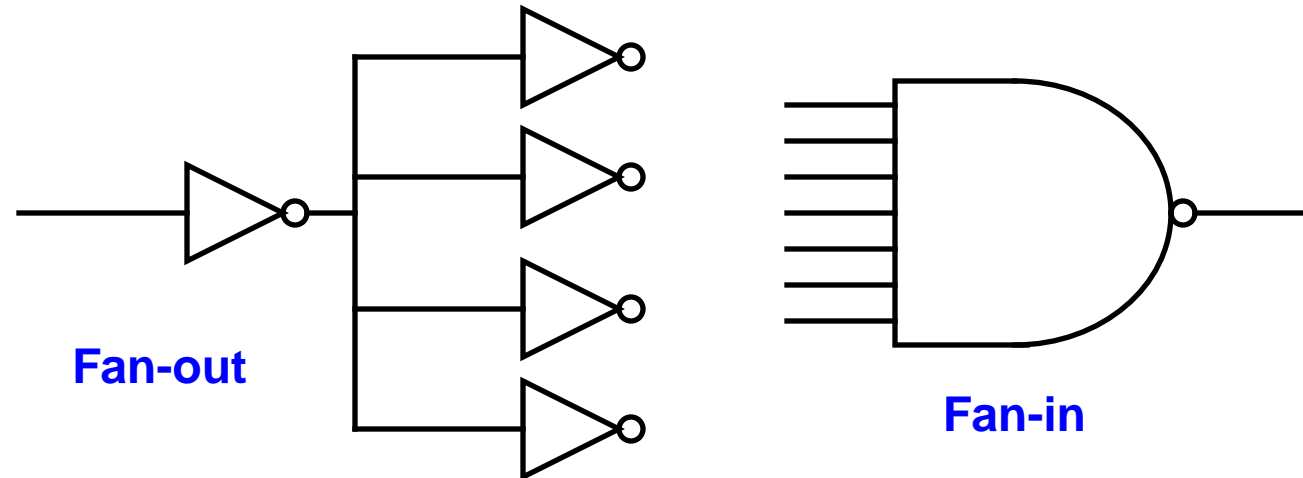


Properties of CMOS

- **Full rail-to-rail swing -> high noise margins**
- **Logic levels not dependent upon the relative device sizes -> transistors can be minimum size -> ratioless**
- **Always a path to V_{dd} or GND in steady state -> low output impedance (output resistance in k Ω range) -> large fan-out (albeit with degraded performance)**
- **Extremely high input resistance (gate of MOS transistor is near perfect insulator) -> nearly zero steady-state input current**
- **No direct path steady-state between power and ground -> no static power dissipation**
- **Propagation delay function of load capacitance and resistance of transistors**



Capacitive Load, etc.

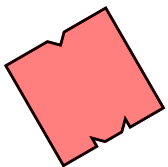


Fan-out: number of gates connected to the output of the driving gate

- Gates with large fan-out are slower

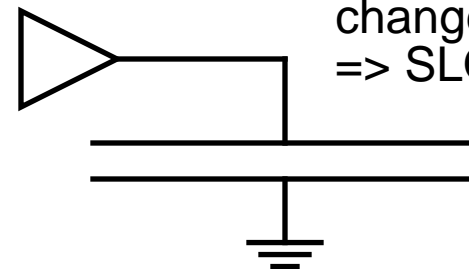
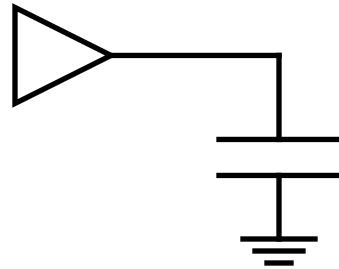
Fan-in: the number of inputs to the gate

- Gates with large fan-in are bigger and slower



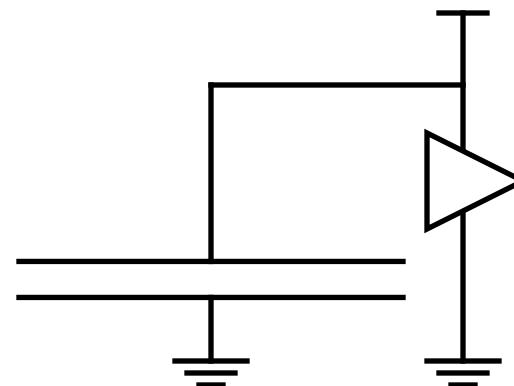
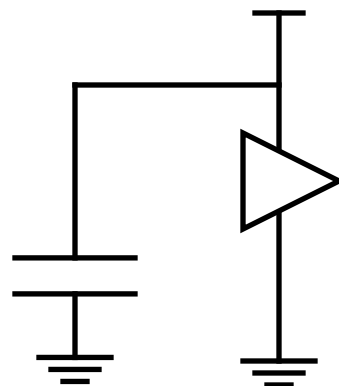
Aside: is capacitance all bad?

Slows down output ...



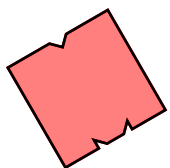
Bigger capacitor,
more charge to
change voltage
=> SLOWER

... but stabilizes power supply

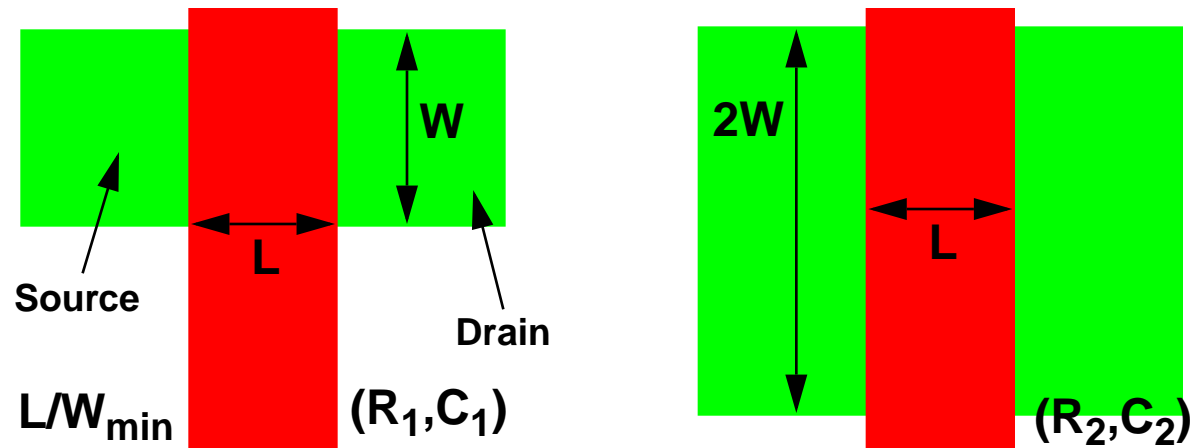


Bigger capacitor,
more charge to
change voltage
=> more stable
power-supply
voltage levels

Capacitors are *de facto* frequency filters ...
can be a good thing (“bypass caps”)



Transistor Sizing I

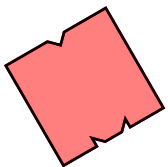


The electrical characteristics of transistors determine the switching speed of a circuit






- Need to select the aspect ratios $(W/L)_n$ and $(W/L)_p$ of every FET in the circuit

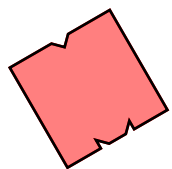
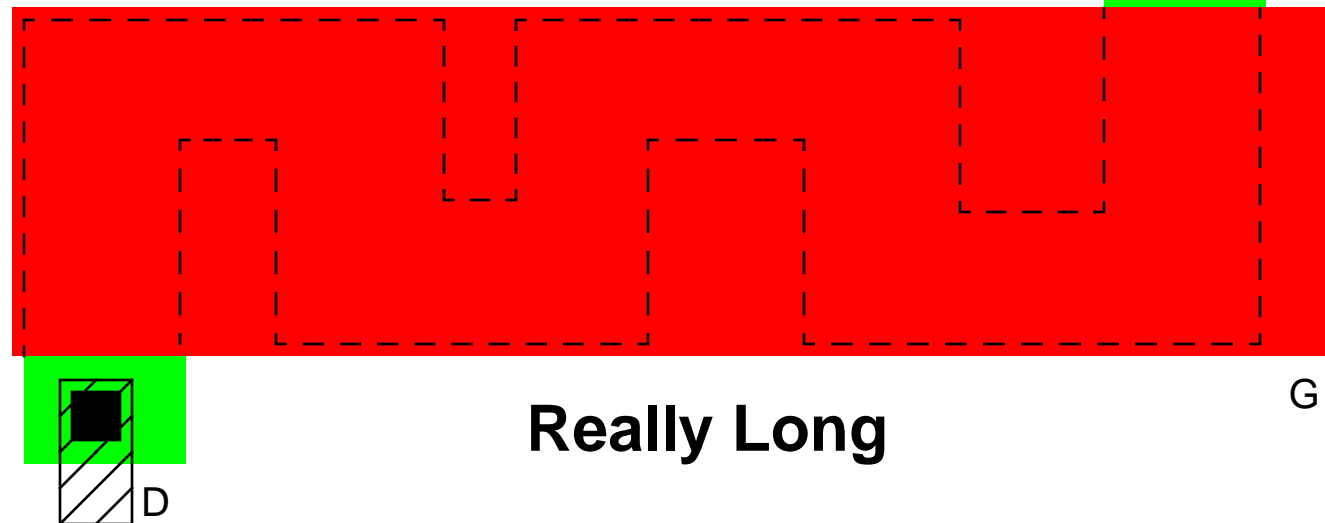
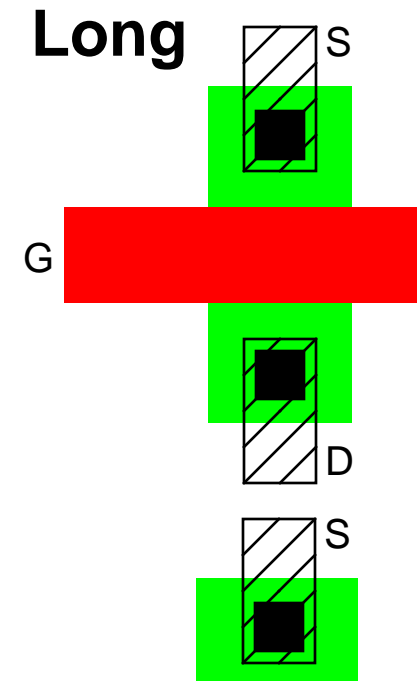
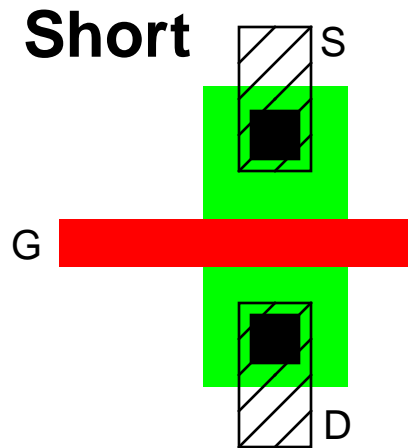
Define *Unit Transistor* (R_1, C_1)

- L/W_{\min} \rightarrow highest resistance
- $R_2 = R_1 \div 2$ and $C_2 = 2 \cdot C_1$
- Separate nFET and pFET unit transistors



Long MOSFETs

-  poly
-  metal
-  active
-  n-well
-  via



Wide MOSFETs

poly

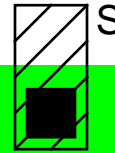
metal

active

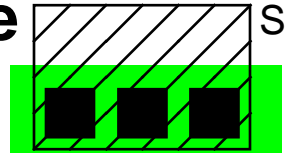
n-well

via

Narrow



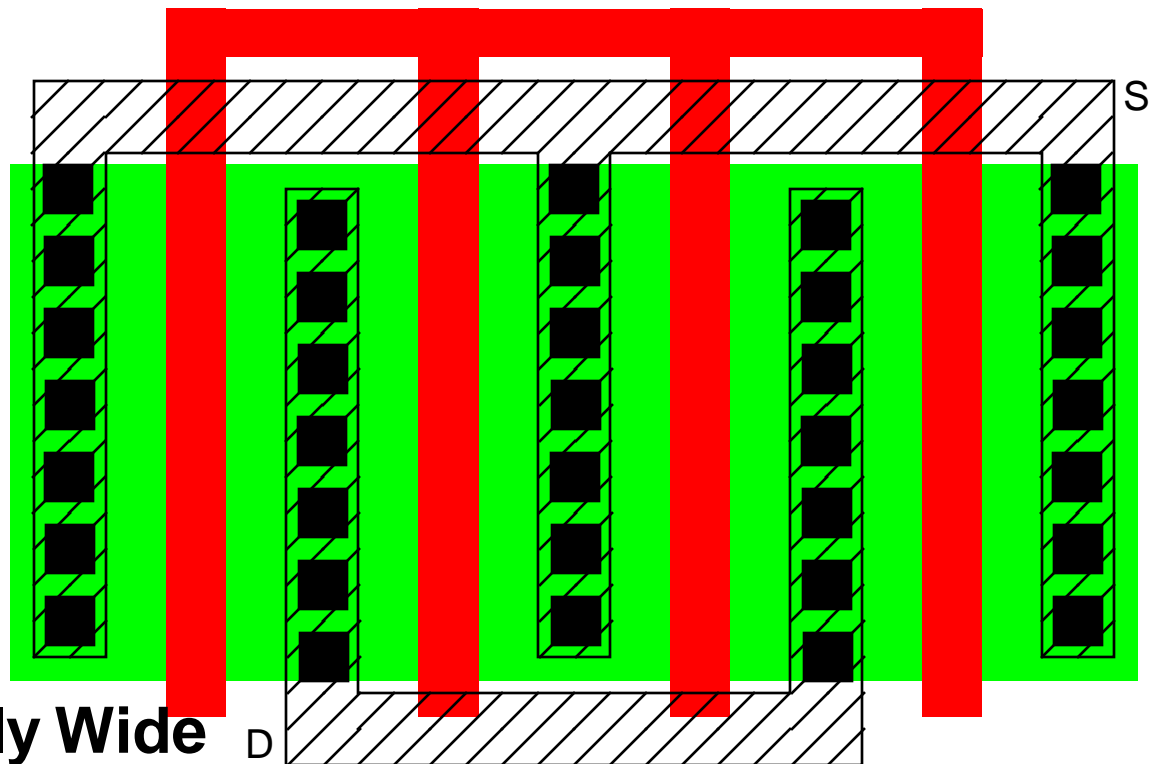
Wide



G

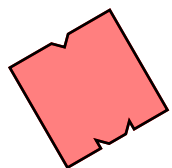
D

D



Really Wide

D



Transistor Sizing II

Resistance of MOSFET:

$$R_n = \frac{1}{\mu_n C_{ox} (V_{GS} - V_{Tn})} \left(\frac{L}{W} \right)$$

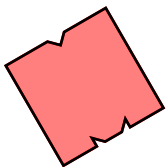
- Increasing W decreases the resistance;
allows more current to flow

$$\text{Oxide capacitance } C_{ox} = \epsilon_{ox} / t_{ox} \text{ [F/cm}^2\text{]}$$

$$\text{Gate capacitance } C_G = C_{ox} WL \text{ [F]}$$

$$\text{Transconductance } \beta_n = \mu_n C_{ox} \left(\frac{W}{L} \right) = k'_n \left(\frac{W}{L} \right)$$

(units [A/V²])



Transistor Sizing II

nFET vs. pFET

$$R_n = \frac{1}{\beta_n (V_{DD} - V_{Tn})}$$

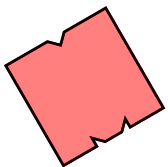
$$\beta_n = \mu_n C_{ox} \left(\frac{W}{L} \right)_n$$

$$R_p = \frac{1}{\beta_p (V_{DD} - |V_{Tp}|)}$$

$$\beta_p = \mu_p C_{ox} \left(\frac{W}{L} \right)_p$$

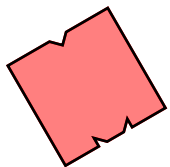
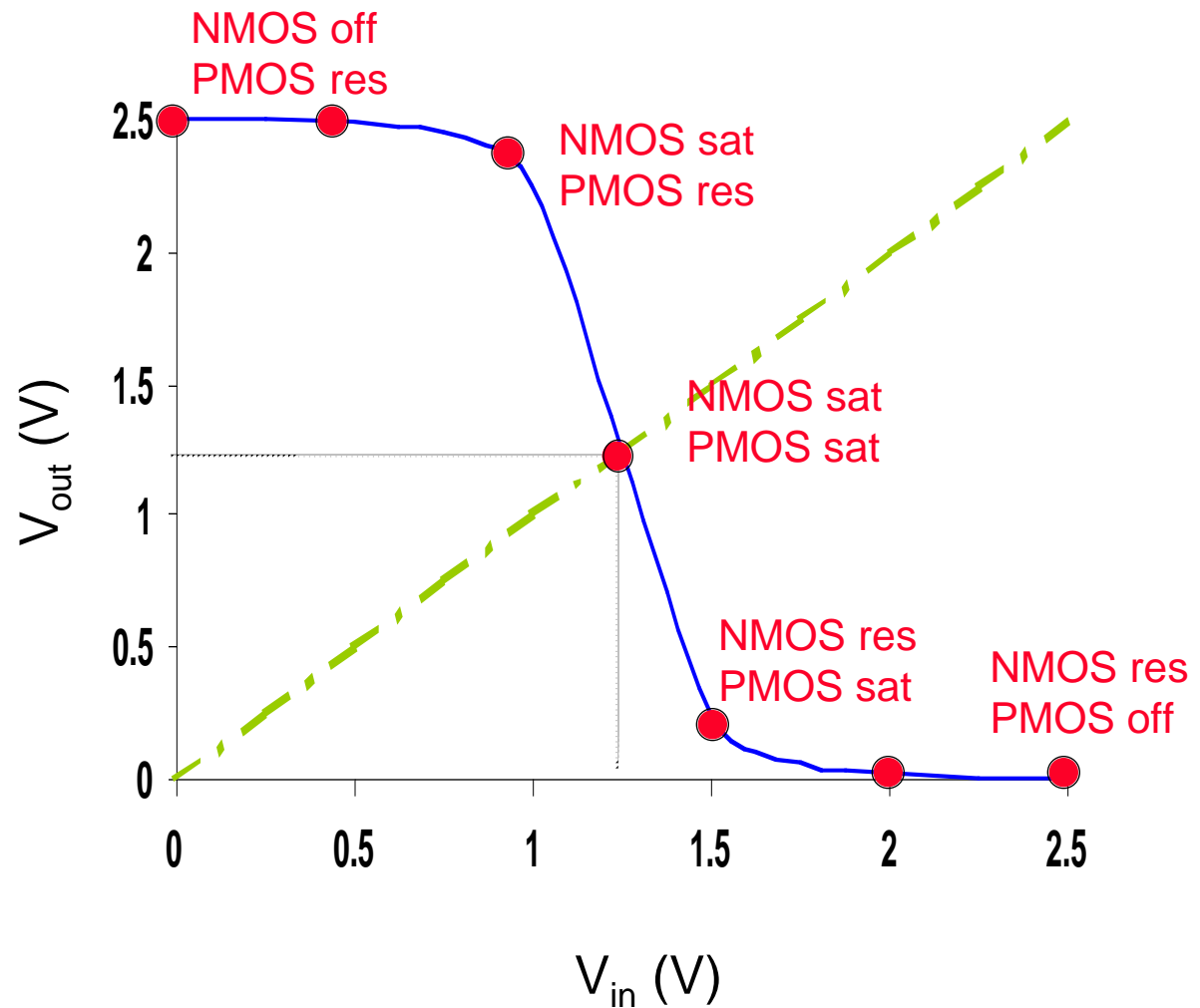
$$\frac{\mu_n}{\mu_p} = r \quad \text{Typically} \\ (2 \dots 3)$$

(μ is the carrier mobility through device)



Inverter Switching Point

Where $V_{in} = V_{out}$



Inverter Switching Point

At all points $I_{DSn} = I_{DSp}$ (drain currents)

At switching point, $V_{in} = V_{out} = V_{sp}$

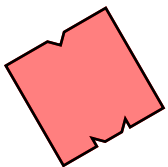
$$\frac{\beta_n}{2}(V_{SP} - V_{Tn})^2 = \frac{\beta_p}{2}(V_{DD} - V_{SP} - V_{Tp})^2$$

$$V_{SP} = \frac{\sqrt{\frac{\beta_n}{\beta_p}} \cdot V_{Tn} + (V_{DD} - V_{Tp})}{1 + \sqrt{\frac{\beta_n}{\beta_p}}}$$

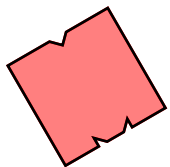
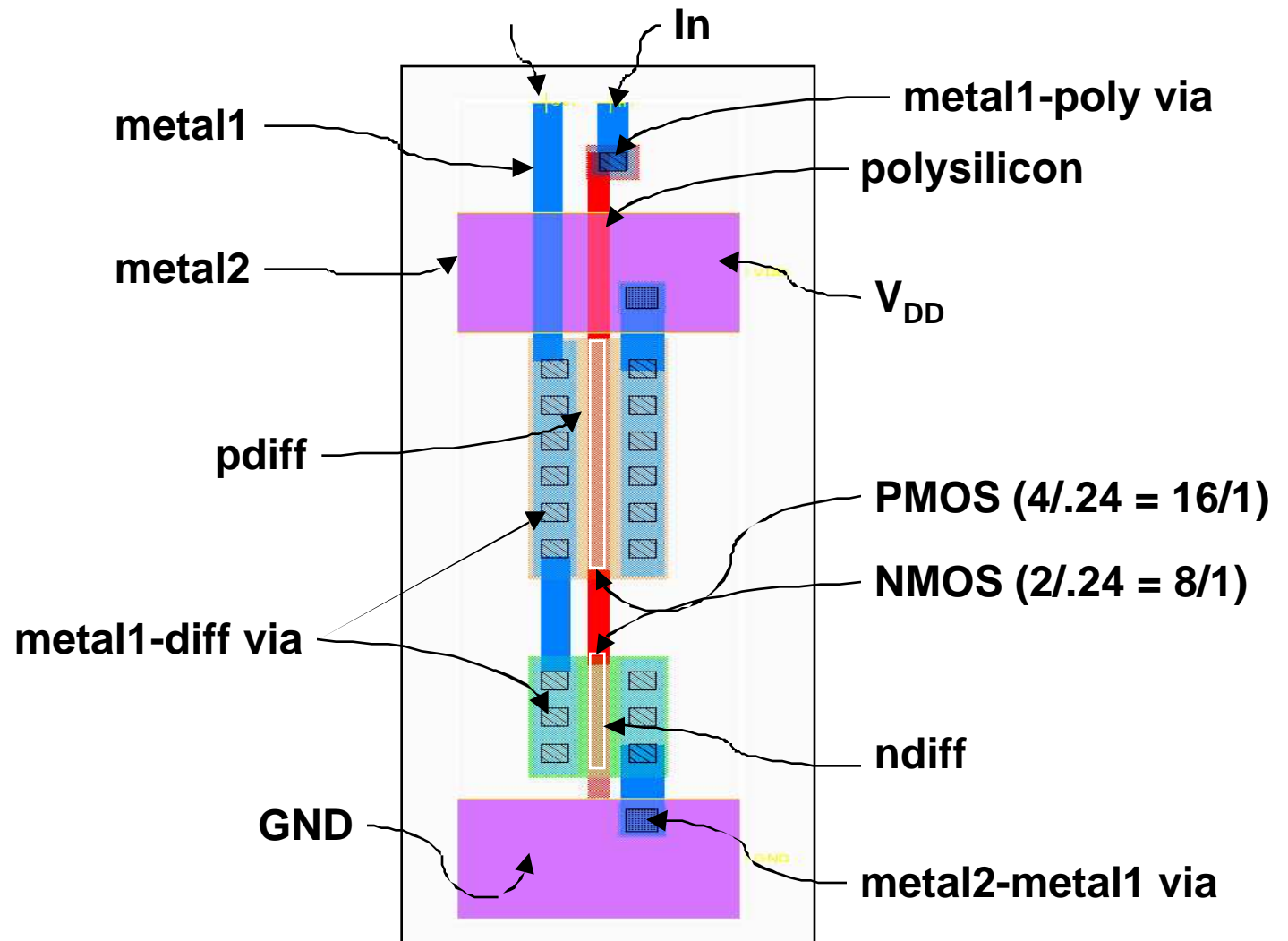
For $V_{sp} = V_{DD}/2$, assuming $V_{Tn} = V_{Tp}$,

$\beta_n = \beta_p \Rightarrow W_p \approx 2-3W_n$

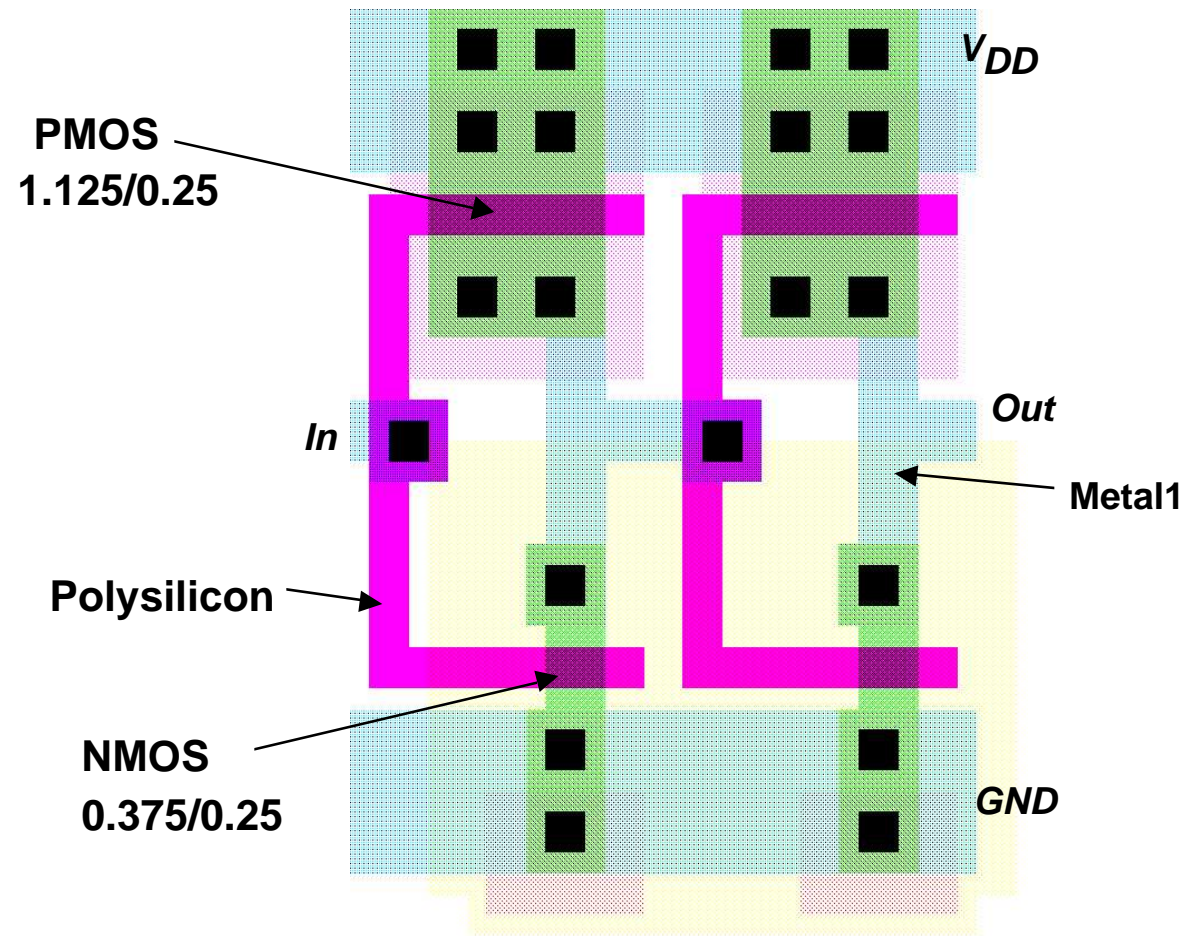
(equal drive currents, equal R_{eff} : $R_n = R_p$)



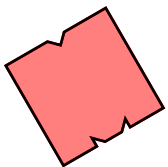
The Result ($W_p = 2W_n$, $.25\mu\text{m}$)



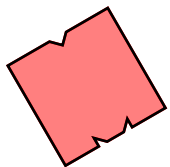
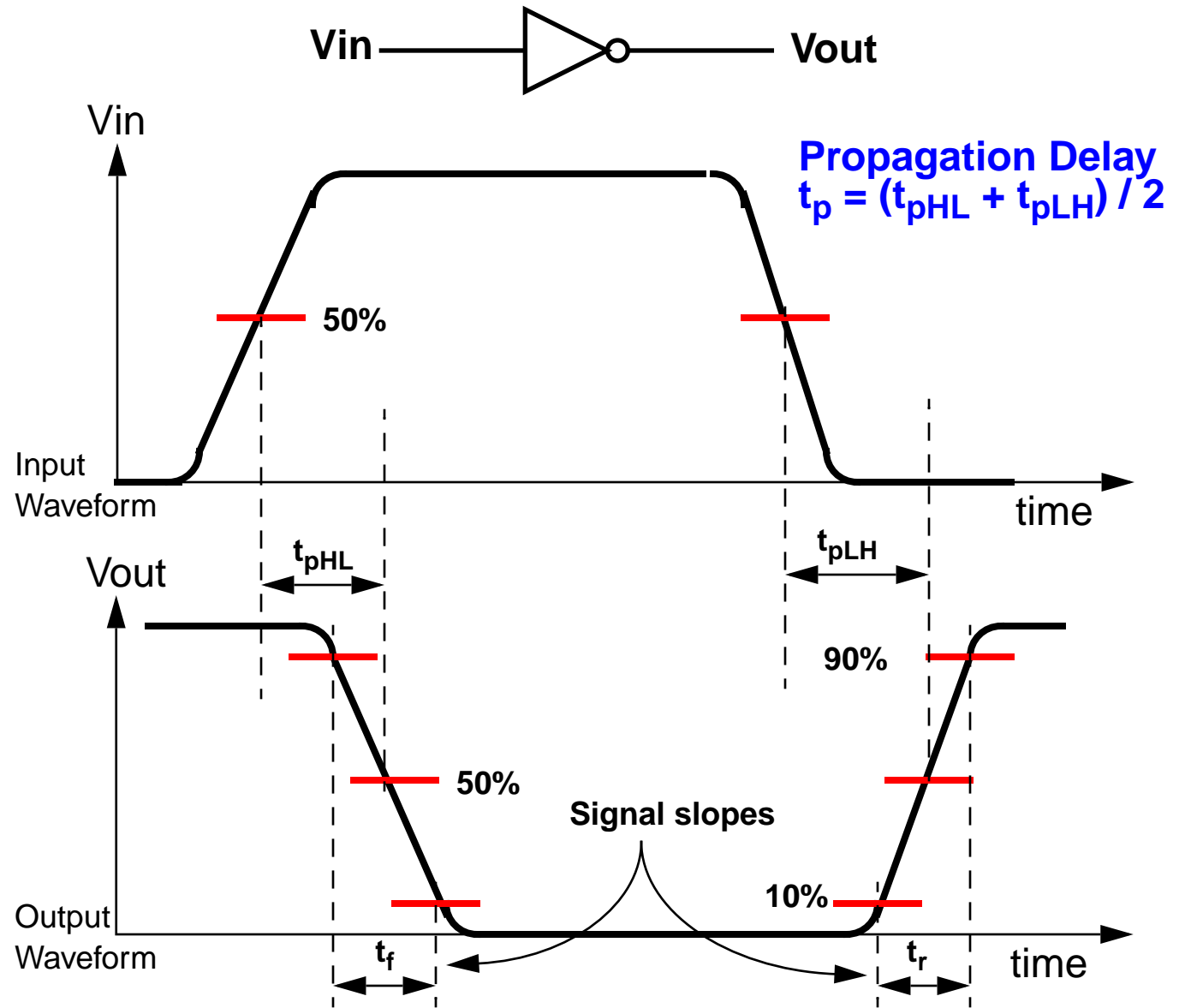
The Result II ($W_p = 3W_n$)



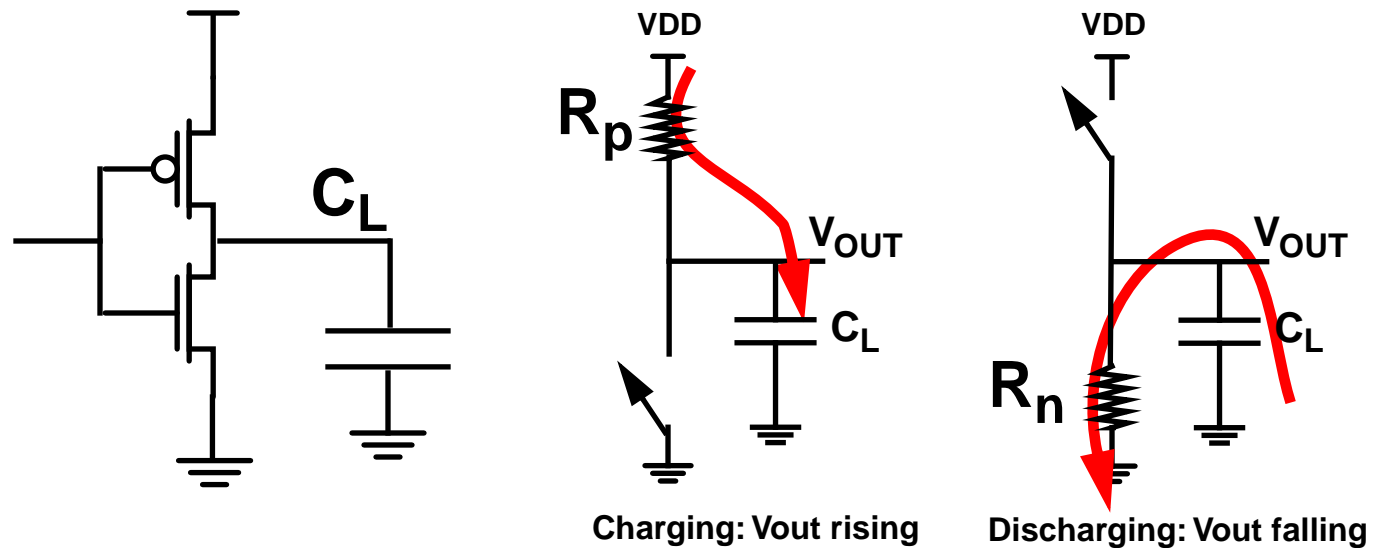
PMOS devices 3x larger than NMOS devices



Delay Definitions

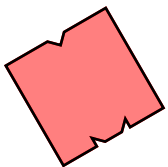


Inverter Switching Delay

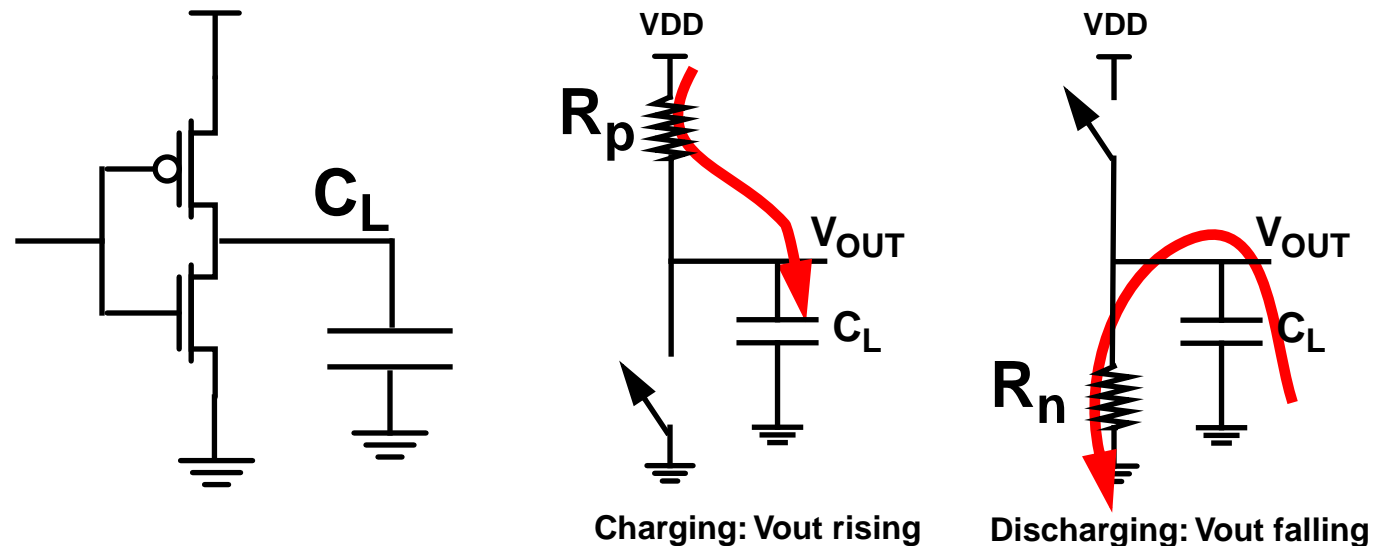


If $(W/L)_p = r(W/L)_n$ then $\beta_n = \beta_p$
(and $R_n = R_p$)
... symmetric inverter

Make pFET bigger (wider) by factor of r



Inverter Switching Delay

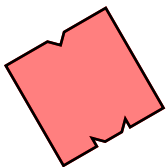


$$t_{pLH} = \ln(2) R_p C_L = 0.69 R_p C_L$$

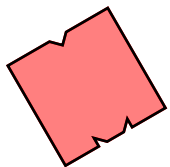
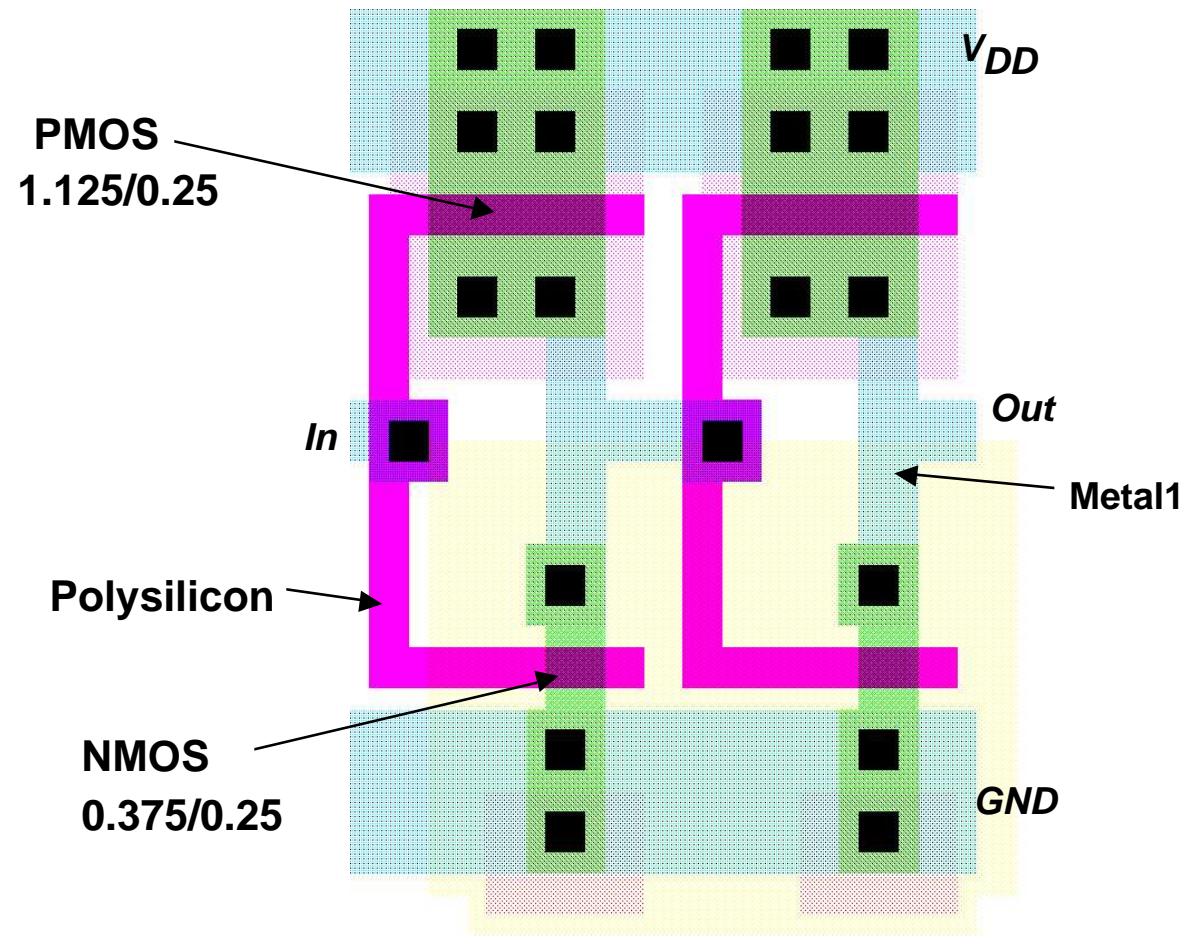
$$t_{pHL} = \ln(2) R_n C_L = 0.69 R_n C_L$$

$$t_p = (t_{pHL} + t_{pLH})/2 = 0.69 C_L (R_n + R_p)/2$$

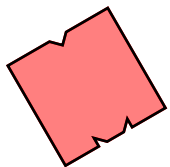
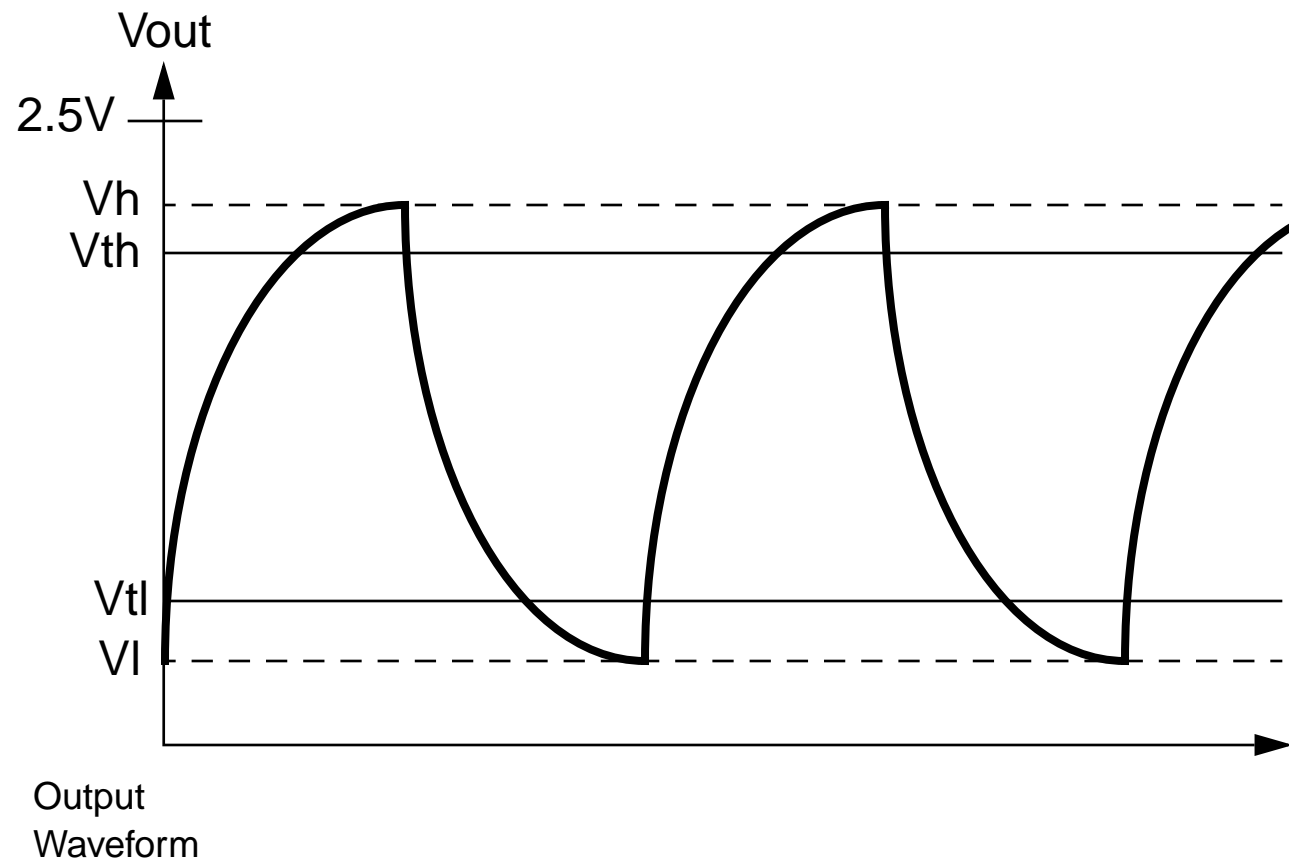
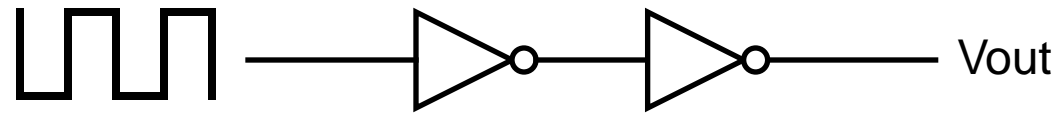
(note: the $\ln(2)RC$ term comes from first-order analysis of simple RC circuit's response to step input ... time for output to reach 50% value)



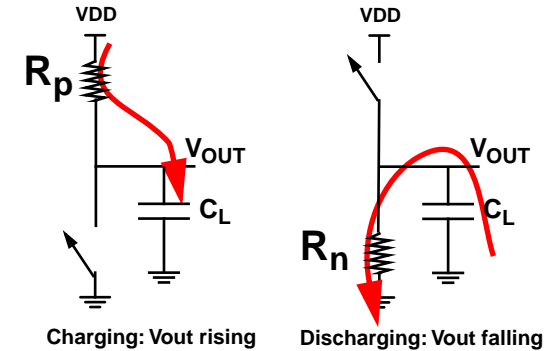
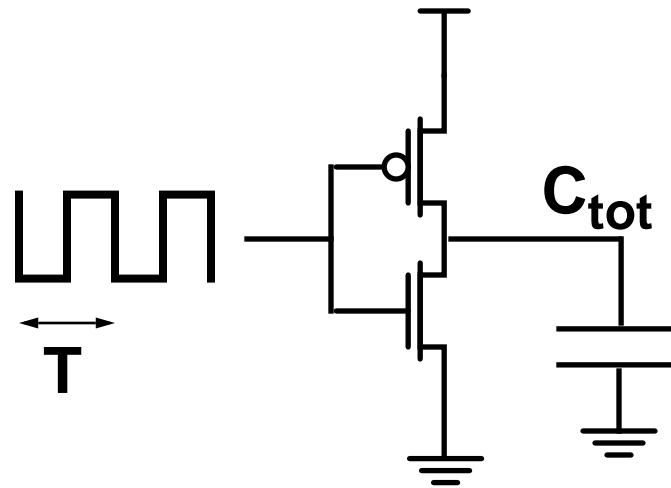
Inverter Pair



Inverter Pair



Dynamic Power Dissipation



$$I_{avg} = \frac{Q_{C_{tot}}}{T} = \frac{V_{DD} \cdot C_{tot}}{T}$$

$$P_{avg} = V_{DD} \cdot I_{avg} = \frac{C_{tot} \cdot V_{DD}^2}{T} = C_{tot} \cdot V_{DD}^2 \cdot f_{CLK}$$

