ENEE 661 Spring 2013 Homework 1

due date February 7 (Thursday)

1. For curve $\gamma : [t_0, t_f] \to \mathbb{R}^3$, $t \mapsto \gamma(t)$ show that curvature and torsion take the form

$$\kappa = \frac{\|\dot\gamma \times \ddot\gamma\|}{|\dot\gamma|^3}$$

$$\tau = \frac{\dot\gamma \cdot (\ddot\gamma \times \dddot\gamma)}{\|\dot\gamma \times \ddot\gamma\|^2}$$

2. Show that the collection of matrices of the form

$$\begin{pmatrix} 1 & a_{12} & a_{13} & 0 \\ 0 & 1 & a_{23} & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & a_{44} \end{pmatrix}$$

with $a_{44} \neq 0$ is a matrix (Lie) group. Determine a basis for its Lie algebra, and associated structure constants.

3. Show that the smallest Lie algebra of matrices which contains the matrices A_1, A_2:

$$A_1 = \begin{pmatrix} 0 & 1 \\ 0 & 1 \end{pmatrix} \quad ; \quad A_2 = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$$

is four dimensional.

4. Let $t \mapsto \Phi(t)$ be a smooth curve in $Sl(n; \mathbb{R})$, show that we can write

$$\dot{\Phi}(t) = \Phi(t)\, \xi(t)$$

where $\xi(t)$ has zero trace $\forall\, t$.

5. Let $t \longmapsto \Phi(t)$ be a smooth curve in $SE(n; \mathbb{R})$ the special Euclidean group of matrices of the form

$$\left(\begin{array}{c|c} A & b \\ \hline 0 & 1 \end{array}\right)$$

where $A \in SO(n)$, $b \in \mathbb{R}^n$, and there is a row of zero's below A. Show that the associated Lie algebra is made up of matrices of the form

$$\left(\begin{array}{c|c} \Omega & \eta \\ \hline 0 & 0 \end{array}\right) \quad \text{where } \Omega = -\Omega^T \text{ and } \eta \in \mathbb{R}^n$$

For $n = 3$ determine all structure constants in a suitable/natural basis for this Lie algebra, denoted as $se(3; \mathbb{R})$