ENEE 661 Spring 2013 Homework 2
due date: February 14 (Thursday)

1. Read carefully the material on Fundamental Theorem of Calculus, from page 10 to page 14 of Lecture Notes 2(b) available online. Using the HINT on page 14, show that

$$\phi_g^{-\varepsilon}(\phi_f^{-\varepsilon}(\phi_g^{\varepsilon}(\phi_f^{\varepsilon}(x_0))))$$

$$= \phi_{-g}^{\varepsilon}(\phi_{-f}^{\varepsilon}(\phi_g^{\varepsilon}(\phi_f^{\varepsilon}(x_0))))$$

$$= x_0 + \varepsilon^2 [f, g](x_0) + O(\varepsilon^3)$$

Here $[f, g]$ denotes the Jacobi-Lie bracket of vector fields f and g; $\left(\frac{\partial g}{\partial x}\right) f - \left(\frac{\partial f}{\partial x}\right) g$

$\phi_f^{\varepsilon}(x)$ is the flow of the vector field f evaluated at time ε on (initial condition) x.

2. Consider the unicycle and the problem of parking: transport the unicycle from $(x=0, y=0, \theta=\pi/2)$ to $(x=0, y=1, \theta=\pi/2)$. Using MATLAB explore the parking algorithm: drive, steer, reverse drive, reverse steer, ..., to solve this problem. Suppose you cannot reverse drive. Can you solve the parking problem? If yes, what is the difference between the two solutions?

3. Consider kinematic car in pages 16-18 of Lecture Notes 2(b). Verify the bracket relations for general θ.

[OPTIONAL: Explore the parking algorithm for this setting numerically]

4. Read Chapter 1 of the textbook (H. Khalil - Nonlinear Systems, 3rd Edition, 2002) and do the exercises:

1.11 and 1.18

Problem 1.11 requires basic concepts from ENEE 660 System Theory - you may find my notes from fall 2010 of use.

5. Read Appendix A (Mathematical Review) and Appendix B (Contraction Mapping) of the textbook and Lecture Notes 3 online.