ENEE 661 SPRING 2017 Homework 3

due date: February 28, Tuesday (beginning of class)

grace period - March 2

1. Consider the system

$$\frac{d}{dt}\begin{bmatrix} T & M_1 & M_2 & \gamma \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} T & M_1 & M_2 & \gamma \\ 0 & 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 0 & -\nu u & -\nu v & \nu \\ \nu u & 0 & 0 & 0 \\ \nu v & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

with $[T(0)\ M_1(0)\ M_2(0)] = I_3$ the 3×3 identity matrix, $\gamma(t)$ defines a curve in $\mathbb{R}^3$ and ν, u, v are scalar control functions (of time). What is the classical curvature κ of γ as a function of time?

HINT: Look for an expression involving ν, u and v.

2. Let $s \mapsto \gamma(s)$, $a < s < b$, $\gamma(s) \in \mathbb{R}^3$ be a unit speed curve. Let $s \mapsto T(s)$ denote the associated unit tangent vector, varying with s.

(i) Under what conditions is the curve $s \mapsto T(s)$ a regular curve?

(ii) Suppose κ and τ are the curvature and torsion functions of the curve γ. Suppose $\kappa(s) > 0$. Let $\bar{s}$ denote the unit speed parametrization of T and let $\bar{\kappa}$ denote the curvature function of T. Show that

$$\bar{\kappa} = \sqrt{1 + (\epsilon/\kappa)^2}$$

3. Show that the collection of matrices of the form

$$\begin{pmatrix} 1 & a & b & 0 \\ 0 & 1 & c & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & d \end{pmatrix}$$

with $d \neq 0$ is a matrix Lie group. Determine a basis for its Lie algebra and associated structure constants.

4. Show that the <u>smallest</u> Lie algebra of 2×2 matrices which contains the matrices A_1, A_2:

$$A_1 = \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}; \quad A_2 = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$$

is <u>four dimensional</u>.

5 Show that for any fixed square $n\times n$ matrix P, the set $\{X : PX + X'P = 0\}$ of $n\times n$ matrices forms a Lie algebra.

~~4~~.

6. Give a precise and careful argument as to why the sequence of numbers

$$\{S_n : n \geq 1\}$$

defined by

$$S_n = \sum_{k=1}^{n} \frac{1}{\sqrt{k}} \qquad n = 1, 2, \ldots$$

is NOT convergent.

7 Can you use Banach iteration method to solve the equation

$$\sin(a \cos(bx)) = x \, ?$$

Under what hypotheses? <u>Hint</u> : Consider a, b.

8 Read section 5 (starting in page 615) of Roger HOWE's paper.
Then go to sections 3 and 4