Homework Set #7 ENEE 664 SPG 2004 SATURDAY
(to be returned: 04.17.04)

1. In Lecture Notes 8, page 4, example 1, compute ρ.

2. In Lecture Notes 8, page 5, example 2, do a numerical comparison (using MATLAB) of the stated algorithm with Newton's algorithm, for several values of $b > 0$. Explain and discuss your data.

3. The second Fréchet derivative of $T: X \to Y$ is $D^2T(x)$ defined by

$$D^2T(x)(h,k) = \left.\frac{d^2 T}{ds\,dt}(x + th + sk)\right|_{\substack{s=0\\t=0}}$$

where $h, k \in X$.

$D^2T(x)(h,h)$ is called the second variation of T with increment h

For the function $J[x] = \int_{t_1}^{t_2} L(t, x(t), \dot{x}(t))\,dt$ defined on differentiable curves $x(t)$ with fixed end points $x(t_1) = x_1$ and $x(t_2) = x_2$, show that the second variation can be written as:

$$D^2J[x](h,h) = \int_{t_1}^{t_2} \left(P(t)\,h^2(t) + Q(t)\,\dot{h}^2(t)\right)dt .$$

Write the functions $P(t)$ and $Q(t)$. State clearly any assumptions on differentiability etc.

4. Let X be a Banach space. Let $A : X \to X$ be a bounded linear operator. Suppose $\|A\| = a < 1$. Use the contraction mapping theorem to show that,

$(\mathbb{1} - A)$ is invertible, and,

$$\|(\mathbb{1} - A)^{-1}\| < \frac{1}{1-a} .$$

5. In problem 2 above, replace the Newton algorithm by

(i) The modified Newton algorithm

$$x_{n+1} = x_n - \lambda_n (DP(x_n))^{-1} P(x_n)$$

where λ_n is a step-size parameter selected according to the Armijo step size rule [This is the Armijo-Newton method — see page 55 of Tits' notes and page 37 of same notes.

(ii) Carry out a numerical comparison with the results of problem 2.
Which of the three algorithms is empirically the fastest?