Circuits Fall 2016

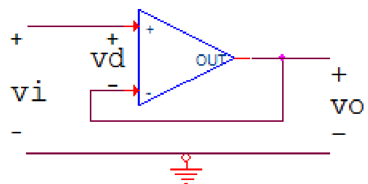
#1. (6 points) For the following circuit

a) (3 points) Assume that the op-amp is ideal (linear, infinite gain, no input current). Find the transfer function vo(s)/vi(s).

b) (3 points) Next assume that the op-amp gain, vo/vd, rather than being infinite, has a first order pole and is given by

$K(s)=K_o/(s+s_o)$

where K_o and s_o are real positive constants. In this case find vo(s)/vi(s) and give its zeros and poles



#2 (7 *points*) The standard form for a degree two bandpass filter is

$$T(s,\zeta) = \frac{2\zeta s}{s^2 + 2\zeta s + 1}$$

where ζ is the damping factor .

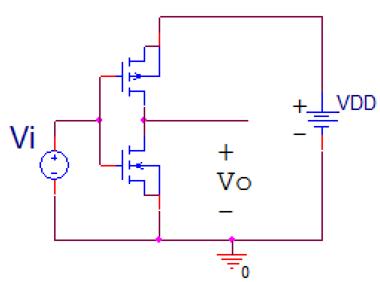
a) (4 points) For sinusoidal excitations (that is, when s=jω) find the peak value of the magnitude $|T(j\omega. \zeta)|$ for fixed ζ.

b) (3 points) Obtain the sensitivity, S_ζ^T, of T(s,ζ) to the damping factor ζ and give its zeroes and poles. Here the sensitivity of a function f to a parameter x is defined as $S_x^f = \frac{\partial f/\partial x}{f/x}$. (Note: the physical meaning of S_x^f is the relative change of f with respect to the relative change of x).

#3) (7 points) For the circuit below assume that when Vo=Vi both transistors are in saturation with the NMOS described by $I_{Dn} = k_n(V_{GS} - V_{th})^2(1+\lambda V_{DS})$ and matching PMOS ($|V_{thp}|=V_{th}$; $\lambda_p = \lambda$) except that $k_p \neq k_n$.

a) (3 points) Show that indeed the two transistors are in saturation when Vo=Vi (recall that saturation means that $V_{GS}-V_{th} < V_{DS}$ for NMOS)

b) (4 points) For V_{th}=1Volt, λ=0.1 and VDD=9Volts, find the ratio k_p/k_n such that Vi=Vo=6Volts.



Solutions Circuits Fall 2016

#1. a) By KVL $v_0 = v_i - v_d$ but $v_d = 0$ by op-amp virtual input connection

$\therefore\ v_0 = v_i$ or $\frac{v_0}{v_i} = 1$

b) as above $v_0 = v_i - v_d$ but $v_0 = K(s) v_d = K(s)\cdot(v_i - v_0)$

$\Rightarrow (1+K) v_0 = +K v_i \Rightarrow \frac{v_0}{v_i} = \frac{+K}{1+K} = \frac{K_0/(s+s_0)}{1+K_0/(s+s_0)} = \frac{K_0}{s+s_0+K_0}$

$\Rightarrow \frac{v_0}{v_i} = \frac{+K_0}{s+s_0+K_0} \Rightarrow$ pole @ $s = -(s_0+K_0)$

zero @ $s = \infty$

#2. We desire ω_m for $\left.d|T(j\omega,\zeta)|/d\omega\right|_{\omega=\omega_m} = 0$ which is $\left.d|T(j\omega,\zeta)|^2/d\omega^2\right|_{\omega^2=\omega_m^2} = 0$

a) $|T(j\omega,\zeta)|^2 = TT^* = \frac{(2\zeta\omega)^2}{(1-\omega^2)^2+(2\zeta\omega)^2} = \frac{(2\zeta)^2\omega^2}{D(\omega^2)}$

$d|T(j\omega,\zeta)|^2/d(\omega^2) = \frac{(2\zeta)^2}{D} - \frac{(2\zeta)^2\omega^2[-2(1-\omega^2)+(2\zeta)^2]}{D^2} =$

$= \frac{(2\zeta)^2}{D^2}\left[(1-\omega^2)^2+(2\zeta)^2\omega^2 - \omega^2[-2(1-\omega^2)+(2\zeta)^2]\right] = \frac{(2\zeta)^2(1-\omega^2)}{D^2}[2\omega^2+1-\omega^2]$

$= \frac{(2\zeta)^2}{D^2}(1+\omega^2)(1-\omega^2) = 0$ @ $\omega_m^2 = 1 \Rightarrow \underline{\underline{\omega_m = 1}}$ (or -1 as $|T|$ is even in ω) (note $D \geq 0\ \forall\omega$)

@ $s = j1$, $T(j\omega_m) = \frac{2\zeta j1}{(-1+2\zeta j\omega+1)} = \frac{j2\zeta}{j2\zeta} = 1 =$ max of $|T(j\omega)|$

b) For S_ζ^T need $\partial T(s,\zeta)/\partial\zeta = \frac{2s}{s^2+2\zeta s+1} - \frac{2\zeta s(2s)}{(s^2+2\zeta s+1)^2} = \frac{2s^3+2s}{(s^2+2\zeta s+1)^2}$

$\therefore\ S_\zeta^T = \frac{\partial T/\partial\zeta}{T/\zeta} = \frac{\zeta}{2\zeta s/(s^2+2\zeta s+1)} \cdot \frac{2s(s^2+1)}{(s^2+2\zeta s+1)^2} = \frac{s^2+1}{s^2+2\zeta s+1}$; zero @ $s = \pm j1$, poles $= -\zeta(1\pm\sqrt{1-1/\zeta^2})$

$\therefore$ the sensitivity is zero at the peak magnitude

#3. a) @ $v_0 = v_i$, $v_{GS_n} - V_{th} = v_i - V_{th} < v_0$; $-v_{GS_p} - (-V_{th}) = -(v_i - V_{DD} - V_{th}) > -(v_0 - V_{DD})$

$\Rightarrow -v_i + V_{th} > -v_0$

$\Rightarrow v_i - V_{th} < v_0$ for PMOS

$\Rightarrow$ NMOS & PMOS both in saturation

b) Use $i_{D_n} = -i_{D_p}$

$\Rightarrow k_n(v_i - V_{th})^2(1+\lambda v_0) = k_p(-(v_i - V_{DD}) - V_{th})^2(1-\lambda[v_0 - V_{DD}])$

$\Rightarrow \frac{k_p}{k_n} = \frac{(v_i - V_{th})^2(1+\lambda v_i)}{(V_{DD} - v_0 + V_{th})^2(1-\lambda[v_i - V_{DD}])} = \frac{(6-1)^2(1+0.6)}{(9-6-1)^2(1+0.3)} = \frac{25\times1.6}{4\times1.3} = \frac{40}{5.2} = 7.69$

$= \frac{100}{13}$

$= 7.69$