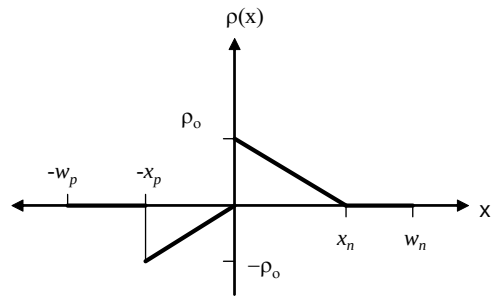
Problem 1 - Diode

Consider a silicon diode with the charge distribution illustrated on the right. Make LARGE labeled sketches of:



a) (4 pts) the electric field *E(x).* (Label the field at x = 0!)

b) (3 pts) the potential $\phi(x)$ - for this plot, assume that $\phi(0) = 0$

Notice first that $x_n = x_p$ because the charges on either side of the junction must balance out.

1a) **for x > 0**, $\rho(x) = \rho_0 - \rho_0 \dfrac{x}{x_n}$

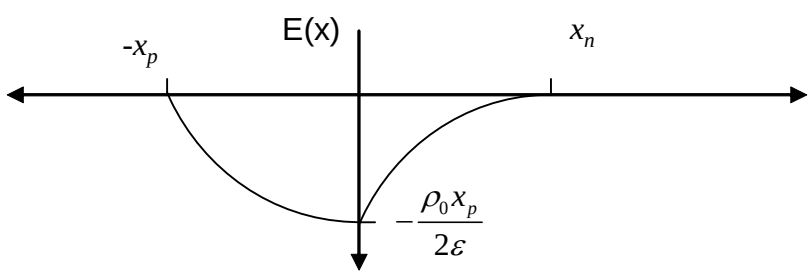
We integrate once and incorporate the zero electric field boundary condition at x_n:

$$E(x) = \frac{\rho_o}{\varepsilon}\left(x - \frac{x^2}{2x_n}\right) + C \quad \text{where} \quad C = -\frac{\rho_0 x_n}{2\varepsilon}$$

for x < 0, $\rho(x) = \rho_0 \dfrac{x}{x_p}$.

We integrate once and incorporate the zero electric field boundary condition at $-x_p$:

$$E(x) = \frac{\rho_0}{2\varepsilon x_p} x^2 + C \quad \text{where} \quad C = -\frac{\rho_0 x_p}{2\varepsilon}$$



1b)

Since $\dfrac{d\phi(x)}{dx} = -E(x)$, we flip the sign of the electric field and integrate:

for x > 0, $\dfrac{d\phi(x)}{dx} = -\dfrac{\rho_o}{\varepsilon}\left(x - \dfrac{x^2}{2x_n}\right) + \dfrac{\rho_0 x_n}{2\varepsilon}$ so,

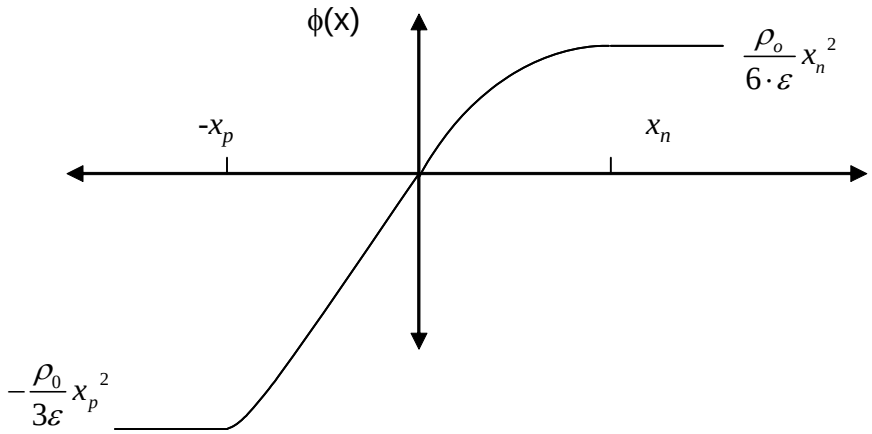
$$\phi(x) = -\frac{\rho_o}{2\varepsilon}x^2 + \frac{\rho_o}{2\cdot 3\cdot \varepsilon x_n}x^3 + \frac{\rho_0 x_n}{2\varepsilon}x + C \quad \text{since } \phi(0) = 0, C = 0.$$

$$\boxed{\phi(x) = -\frac{\rho_o}{2\varepsilon}x^2 + \frac{\rho_o}{2\cdot 3\cdot \varepsilon x_n}x^3 + \frac{\rho_0 x_n}{2\varepsilon}x} \quad \text{and} \quad \boxed{\phi(x_n) = \frac{\rho_o}{6\cdot \varepsilon}x_n^{\,2}}$$

for x < 0, since $\frac{d\phi(x)}{dx} = -E(x)$, $\frac{d\phi(x)}{dx} = -\frac{\rho_0}{2\varepsilon x_p}x^2 + \frac{\rho_0 x_p}{2\varepsilon}$ Integrating once,

$\phi(x) = -\frac{\rho_0}{2\cdot 3\cdot \varepsilon x_p}x^3 + \frac{\rho_0 x_p}{2\varepsilon}x + C$ since $\phi(0) = 0, C = 0$.

$\boxed{\phi(x) = -\frac{\rho_0}{2\cdot 3\cdot \varepsilon x_p}x^3 + \frac{\rho_0 x_p}{2\varepsilon}x}$ and $\boxed{\phi(-x_p) = \frac{\rho_0}{6\cdot\varepsilon}x_p^{\,2} - \frac{\rho_0}{2\varepsilon}x_p^{\,2} = \frac{\rho_0}{\varepsilon}x_p^{\,2}\left(\frac{1}{6}-\frac{1}{2}\right) = -\frac{\rho_0}{3\varepsilon}x_p^{\,2}}$



Notice the asymmetry in curvatures and potentials in the quasistatic regions.

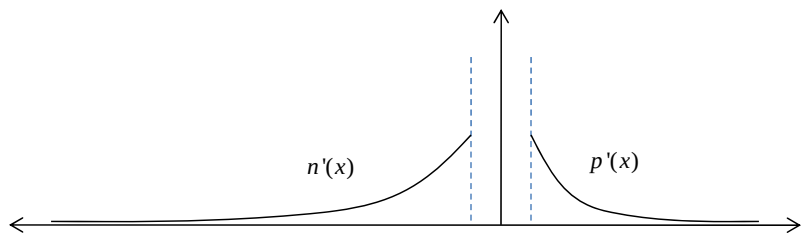
Problem #2 – Diffusion Capacitance

Small signal capacitance is a measure of the change in stored charge needed for a small change in voltage across the device. In the diode, the *diffusion* capacitance is based on the charge found in the quasineutral regions of a forward-biased diode carrying current. Most formulations of diffusion capacitance utilize a p^+n diode where a simple form of the charge distribution can be assumed. In this case, we have a pn diode with equal doping on each side and device widths that are much larger than the minority carrier diffusion length.

a) (3 pts) Assuming forward bias operation, sketch and label the **excess** minority carrier concentration plot for the diode. LARGE PLOT PLEASE. (p-type on the left and n-type on the right).
b) (4 pts) Find a simplified expression for the total charge stored in the quasi-neutral regions (do not leave any integrals in the equation).

In this problem, the excess minority carrier concentrations are non-zero on both sides of the diode (forward biased) and because the width >> diffusion length, the minority carrier concentration is a simple decaying exponential. To find the capacitance we calculate the total charge and then differentiate with respect to the voltage v_D.
Although the depletion region-edge excess minority concentrations are the same, the decays are different due to different L_h and L_e values. First we find the total charge by finding area under the curves on both sides.

We note that the *excess* minority carrier concentration on each edge of the depletion region is: $p_{n0} = \frac{n_i^2}{N_D}\left(e^{\frac{V_D}{V_T}} - 1\right)$ for the n-type region and $n_{p0} = \frac{n_i^2}{N_A}\left(e^{\frac{V_D}{V_T}} - 1\right)$ for the p-type region.



$$Q = Aq\frac{n_i^2}{N_D}\left(e^{\frac{v_D}{V_T}} - 1\right)\int_{x_n}^{\infty} e^{\frac{-(x-x_n)}{L_h}}dx + \left|A(-q)\frac{n_i^2}{N_A}\left(e^{\frac{v_D}{V_T}} - 1\right)\int_{-\infty}^{-x_p} e^{\frac{(x+x_p)}{L_e}}dx\right|$$

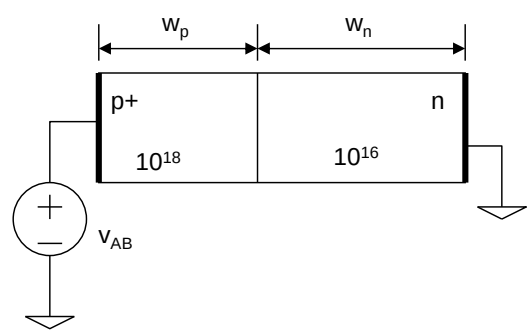
$$Q = \frac{Aqn_i^2}{N_D}\left(e^{\frac{v_D}{V_T}} - 1\right)\left(-L_h e^{\frac{-(x-x_n)}{L_h}}\Big|_{x_n}^{\infty}\right) + \frac{Aqn_i^2}{N_A}\left(e^{\frac{v_D}{V_T}} - 1\right)\left(L_e e^{\frac{(x+x_p)}{L_e}}\Big|_{-\infty}^{-x_p}\right)$$

$$Q = \frac{Aqn_i^2}{N_D}\left(e^{\frac{v_D}{V_T}} - 1\right)(-L_h)(0-1) + \frac{Aqn_i^2}{N_A}\left(e^{\frac{v_D}{V_T}} - 1\right)L_e(1-0)$$

$$\boxed{Q = Aqn_i^2\left(e^{\frac{v_D}{V_T}} - 1\right)\left(\frac{L_h}{N_D} + \frac{L_e}{N_A}\right)}$$

Problem #3 – 6 pts – Depletion Region

Consider the p+ n diode shown to the right.



a) (3 pts) In this diode, at what bias level v_{AB} will the n-region become fully depleted? Assume that breakdown will not happen here.

b) (3 pts) If E_{crit} is the maximum electric field before breakdown occurs, what would be the maximum voltage we can apply at v_{AB}? Would increasing the doping levels on both side increase or decrease this v_{AB} value?

Part a) Because the depletion region grows with reverse bias, vAB should be negative. Given the equation for the depletion region width on one side of a junction (p-side) , we

reconfigure the equation for the n-side $w_{depl} = \sqrt{\frac{\varepsilon(\phi_b)}{q}\frac{N_A}{N_D(N_A+N_D)}}$, and with an applied voltage: $w_{depl} = \sqrt{\frac{\varepsilon(\phi_b - v_{AB})}{q}\frac{N_A}{N_D(N_A+N_D)}}$

Then we solve for v_{AB} for the case where $w_{depl} = w_n$.

$$w_n^{\,2}\frac{q}{\varepsilon}\frac{N_D(N_A+N_D)}{N_A} = \phi_b - v_{AB}$$

$$\boxed{v_{AB} = \phi_b - w_n^{\,2}\frac{q}{\varepsilon}\frac{N_D(N_A+N_D)}{N_A}}$$ note that $\phi_b = V_T \ln\left(\frac{N_A N_D}{n_i^{\,2}}\right)$

Part b) Likewise, we find the peak electric field using the formula provided except with an applied voltage.

$$E_{peak} = \sqrt{\frac{2q(\phi_b - v_{AB})}{\varepsilon}\frac{N_A N_D}{N_A+N_D}}$$

$$E_{crit}^{\,2}\frac{N_A+N_D}{N_A N_D}\frac{\varepsilon}{2q} = \phi_b - v_{AB}$$

$v_{AB} = \phi_b - E_{crit}^{\,2}\frac{N_A+N_D}{N_A N_D}\frac{\varepsilon}{2q}$ Although ϕ_b increases slightly with doping, the second term (which should be much bigger than ϕ_b) is roughly inversely proportional to doping.