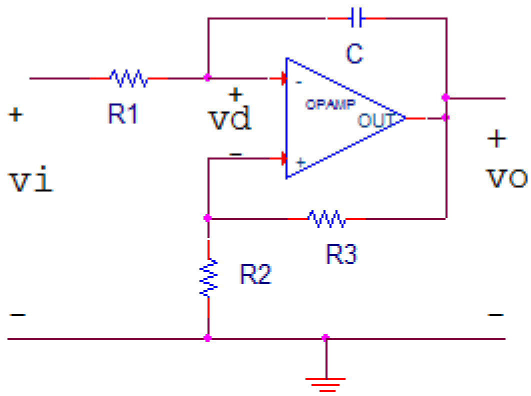
Circuits Fall 2017

#1. (6 points) For the following circuit assume that the op-amp is ideal (linear, zero input currents and zero input difference voltage=vd=0).



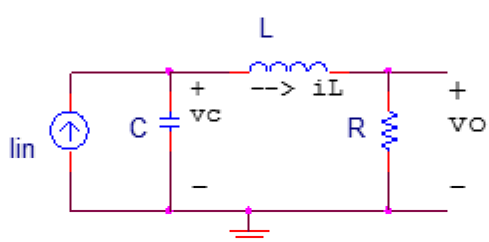
a) (3 points) Find the transfer function vo(s)/vi(s) when R1=R2=R3=R and give its zeros and poles.

b (3 points) Discuss stability of the circuit, including conditions and nature.

#2 (7 *points*) The following circuit is described by the equations

$$\begin{bmatrix} C & 0 \\ 0 & L \end{bmatrix} \frac{dx}{dt} = \begin{bmatrix} 0 & -1 \\ 1 & -R \end{bmatrix} x + \begin{bmatrix} 1 \\ 0 \end{bmatrix} \text{Iin}$$

$$v_0 = [0 \quad R]x$$



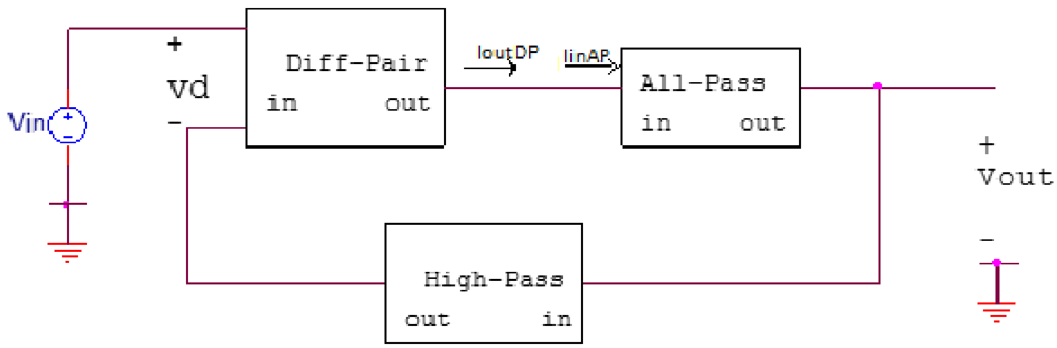
a) (2 points) Give x in terms of voltage and current variables labelled in the circuit.

b) (3 points) Give the transfer function vo/Iin(s).

c) (2 points) For C ,L, R all positive, discuss if this is a low-pass, high-pass or band-pass circuit.

#3) (7 points) The following is a sectioned circuit diagram for a feedback circuit consisting of a differential pair, an all-pass circuit, and a high-pass feedback circuit.

The differential pair is described by IoutDP/vd = Gm, the all-pass circuit is described by voutAP/IinAP(s) = [s-a]/[s+a], and the high-pass feedback by voutHP/vinHP(s) = Cs; all of C, Gm, and a are positive.



a) (3 points) Find the transfer function Vout/Vin(s).

b) (2 poinra) Show that there is a Gm for which this will be a sinusoidal oscillator; give the Gm and oscillation frequency, fosc.

c) (2 points) If the output of the high-pass section were to become shorted, discuss how that will affect a measurement of Vout in the laboratory.

Circuits Fall 2017 solutions

#1. a) $v_{R_2} = \frac{R_2}{R_2+R_3} v_0 = \frac{1}{2} v_0$, $v_{R_1} = R_1 i = v_i - v_d - v_2 = v_i - \frac{1}{2} v_0 \Rightarrow i = \frac{1}{R}(v_i - \frac{1}{2} v_0)$

$i_c = i = sC v_c = sC[v_d + v_2 - v_0] = sC[\frac{1}{2} v_0 - v_0] = -\frac{1}{2} sC v_0 = \frac{1}{R}(v_i - \frac{1}{2} v_0)$

$\Rightarrow -\frac{RC}{2} s v_0 + \frac{1}{2} v_0 = v_i \Rightarrow \frac{v_0}{v_i} = \frac{1}{(-\frac{RC}{2}s + \frac{1}{2})} = \frac{2}{-RCs+1} = \frac{-2/RC}{s - 1/RC} = \frac{v_0}{v_i}$

$\Rightarrow$ a zero @ ∞, a pole @ $s_p = 1/RC$

b) If $R>0, C>0$ the circuit is unstable having a RHP pole giving impulse response $-\frac{2}{RC} e^{t/RC} 1(t)$

If $R=0$, the input is tied to (the limit when $R\to 0$) $\frac{1}{2} v_0 \Rightarrow v_0 = 2v_i \Rightarrow$ stable. Similarly if $C=0$ as the input current is 0 giving $v_i = \frac{1}{2} v_0 \Rightarrow$ stable

#2. Summing currents at the top of C $\Rightarrow i_{in} = C\frac{dv_c}{dt} + i_L \Rightarrow C\frac{dv_c}{dt} = -i_L + i_{in}$

$\Rightarrow$ 1st eq $\Rightarrow x_1 = v_c$

Summing voltages around loop, C-L-R, $\Rightarrow -v_c + L\frac{di_L}{dt} + R.i_L = 0 \Rightarrow L\frac{di_L}{dt} = v_c - Ri_L$

$\Rightarrow$ 2nd eq $\Rightarrow x_2 = i_L$

a) $\Rightarrow x = \begin{bmatrix} v_c \\ i_L \end{bmatrix}$

b) Taking Laplace transform $s\begin{bmatrix} C & 0 \\ 0 & L \end{bmatrix} X(s) = \begin{bmatrix} 0 & -1 \\ 1 & -R \end{bmatrix} X(s) + \begin{bmatrix} 1 \\ 0 \end{bmatrix} I_{in}$, $V_0 = [0 \quad R] X(s)$

$\Rightarrow X(s) = \left\{ \begin{bmatrix} sC & 0 \\ 0 & sL \end{bmatrix} - \begin{bmatrix} 0 & -1 \\ 1 & -R \end{bmatrix} \right\}^{-1} \begin{bmatrix} 1 \\ 0 \end{bmatrix} I_{in} = \begin{bmatrix} sC & 1 \\ -1 & sL+R \end{bmatrix}^{-1} \begin{bmatrix} 1 \\ 0 \end{bmatrix} I_{in} = \frac{1}{s^2LC + sRC + 1} \cdot \begin{bmatrix} sL+R \\ +1 \end{bmatrix} I_{in}$

$V_0 = [0 \quad R] \begin{bmatrix} sL+R \\ +1 \end{bmatrix} \cdot \frac{1}{LCs^2 + RCs + 1} I_{in} \Rightarrow \frac{V_0}{I_{in}} = \frac{R}{LCs^2 + RCs + 1}$

c) This is a low pass circuit if R, L, C all > 0 as $\left.\frac{V_0}{I_{in}}\right|_{s=j\omega\to\infty} = 0$

#3. a) $i_{outOP} = -G_m[v_{in} - Cs v_{out}] = i_{in AP} = \left(\frac{s+a}{s-a}\right) v_{out} \Rightarrow G_m v_{in} = \left[G_m Cs + \left(\frac{s+a}{s-a}\right)\right] v_{out}$

$\Rightarrow \frac{v_{out}}{v_{in}} = \frac{G_m}{G_m Cs + (\frac{s+a}{s-a})} = \frac{G_m(s-a)}{G_m Cs^2 + (1 - G_m Ca)s + a}$

b) For $G_m Ca = 1$ which is $G_m C = 1/a$, $\frac{v_{out}}{v_{in}} = \frac{G_m(s-a)}{G_m Cs^2 + a} = \frac{\frac{1}{C}(s-a)}{s^2 + \frac{a}{G_m C}} = \frac{\frac{1}{C}(s-a)}{s^2 + a^2}$

$\Rightarrow G_m = 1/Ca \Rightarrow$ pole on $j\omega$ axis @ $\pm\sqrt{-a^2} = \pm ja$, $\omega_{oscillation} = a = 1/G_m C$, $f_{osc} = \frac{\omega_{osc}}{2\pi} = a/2\pi$

c) If high-pass output = short $\Rightarrow$ no feedback $\Rightarrow v_d = v_{in}$ killing oscillations

$\Rightarrow \left(\frac{v_{out}}{v_{in}}\right)\Big|_{C=0} = \frac{G_m(s-a)}{s+a} \Rightarrow$ measure only "damped" responses, no self oscillation