Solutions

ECE Written Qualifying Examination, Summer 2017
Digital Logic

1. (4 points) Boolean Simplification.

   Give an example of a pair of Boolean functions $f(x,y,z), g(x,y,z)$ such that $f(x,y,z)$ has more minterms than $g(x,y,z)$ but the cost of the minimal sum-of-products for f is less than the cost of the minimal sum-of products for g.

f:

x \ yz	00	01	11	10
0	1	1	0	0
1	1	1	0	0

g:

x \ yz	00	01	11	10
0	1	0	0	0
1	0	0	0	0

$f(x,y,z) = \overline{y}$ $\qquad$ $g(x,y,z) = \overline{x}\,\overline{y}\,\overline{z}$

2. (4 points) Boolean Algebra.

   Using Boolean Algebra postulates and theorems prove that

$$\overline{x}\,\overline{y} + xz = (\overline{x}+z)(x+\overline{y})$$

   No credit will be given for solutions that use the truth table method.

$$\begin{aligned}
RHS &= (\overline{x}+z)(x+\overline{y}) \\
&= \overline{x}\cdot x + \overline{x}\,\overline{y} + zx + z\overline{y} && \text{(distributive)} \\
&= 0 + \overline{x}\,\overline{y} + zx + z\overline{y} && \text{(complement)} \\
&= \overline{x}\,\overline{y} + zx + z\overline{y} && \text{(additive identity)} \\
&= \overline{x}\,\overline{y} + zx + z\overline{y}\cdot 1 && \text{(multiplicative identity)} \\
&= \overline{x}\,\overline{y} + zx + z\overline{y}(x+\overline{x}) && \text{(complement)} \\
&= \overline{x}\,\overline{y} + zx + z\overline{y}x + z\overline{y}\,\overline{x} && \text{(distributive)} \\
&= (\overline{x}\,\overline{y} + z\overline{x}\,\overline{y}) + (zx + zx\overline{y}) && \text{(commutative, associative)} \\
&= \overline{x}\,\overline{y} + zx = LHS && \text{(absorption)}
\end{aligned}$$

3. (7 points) Synthesis.

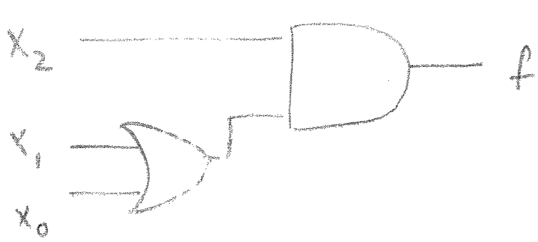
Design a 4-input, 1-output minimal two-level gate combinational network that gets as input a 4-bit number ($x = x_3x_2x_1x_0$) and outputs 0 if $0 \leq x \leq 4$ or $8 \leq x \leq 12$ and outputs 1 otherwise. [Note this circuit will detect the presence of any of the six illegal code gropus in the 5421 code by providing a 1 output.]

X_3	X_2	X_1	X_0	f Out
0	0	0	0	0
0	0	0	1	0
0	0	1	0	0
0	0	1	1	0
0	1	0	0	0
0	1	0	1	1
0	1	1	0	1
0	1	1	1	1
1	0	0	0	0
1	0	0	1	0
1	0	1	0	0
1	0	1	1	0
1	1	0	0	0
1	1	0	1	1
1	1	1	0	1
1	1	1	1	1

X_3X_2 \ X_1X_0	00	01	11	10
00	0	0	0	0
01	0	1	1	1
11	0	1	1	1
10	0	0	0	0

$f = X_2 X_0 + X_2 X_1$

$f = X_2 \cdot X_1 + X_0$ ✓ minimal



4. (5 points) State Diagram.

Draw the state diagram of a Mealy machine having a single input line x, in which the symbols 0 and 1 are applied, and a single output line z. For $i = 1, 2, 3, \ldots$ let x_i denote the i-th input symbol. For $i \geq 3$, the system is to produce an output of 1 coincident with input symbol x_i if the previous 3 bits (from right to left) $(x_i, x_{i-1}, x_{i-2}, x_{i-3})$ form a palindrome (i.e. $(x_i, x_{i-1}, x_{i-2}, x_{i-3}) = (x_{i-3}, x_{i-2}, x_{i-1}, x_i)$). At all other times the system is to output 0. An example of input/output sequences that satisfy the conditions of the system specification is:

$$\begin{array}{lcccccccccccc} x & = & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 0 & 1 \\ z & = & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{array}$$

In the example above, the system produces an output of 1 coincident with the 3-rd input symbol. This occurs since the 1-st, 2-nd and 3-rd input symbols are 1, 0, 1, which is a palindrome.