ENEE 729G: Concentration of Measure for Functions of Random Variables

Description: When studying engineering systems, we oftentimes deal with random variables or their functions. Classical asymptotic results, such as the Law of Large Numbers (LLNs) and the Central Limit Theorem (CLT), provide convenient approximations for the sum of a large number of independent and identically distributed (i.i.d.) random variables. These results tell us that the sum of many i.i.d. random variables is close to its expectation with high probability, indicating that its distribution is concentrated around its mean. It turns out that such concentration of distribution or measure occurs for a large class of *functions* of independent random variables, and there are (by now well-known) bounds on deviations from the expectations or other values. Such bounds are called concentration inequalities. The goal of the proposed course is to introduce the students to some of the key results on concentration inequalities and popular methods that are used in Information Theory, Learning Theory, Random Matrix Theory, and Statistical Mechanics, just to name a few.

Recommended Textbook:

- S. Boucheron, G. Lugosi, and P. Massart, *Concentration Inequalities: A Nonasymptotic Theory of Independence*, Oxford University Press, 2013

References:

- D.P. Dubhashi and A. Panconesi, *Concentration of Measure for the Analysis of Randomized Algorithms*, Cambridge University Press, 2012.
- S. Boucheron, G. Lugosi, and O. Bousquet, "Concentration inequalities," Advanced Lectures on Machine Learning, LNCS, 3176:208-240, 2004.

Course Outline: I will follow the organization of the recommended book (*Concentration Inequalities: A Nonasymptotic Theory of Independence* by S. Boucheron, G. Lugosi, and P. Massart) with some reshuffling. Each topic will be covered in 1-3 lectures.

1. Introduction
2. Basic inequalities
3. Variance bound
4. Information inequalities
5. Logarithmic Sobolev (log-Sobolev) inequalities
6. The entropy method
7. The transportation method
8. Concentration and isoperimetric inequalities
9. Suprema of empirical processes

Prerequisite: ENEE 620

Course Organization and grading: There will be bi-weekly homework assignments, a midterm and a regularly scheduled final exam.