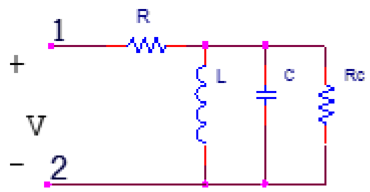
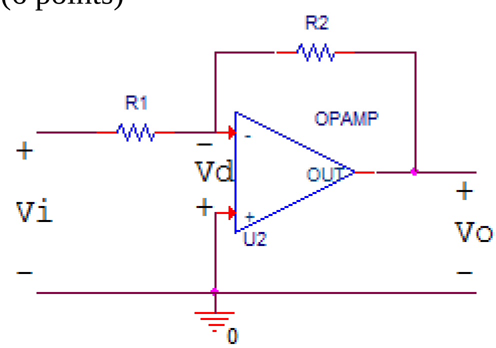
Circuits Spring 2017

#1. (7 points)



a) For the above circuit find the input admittance, $y_{in}(s)$, seen between nodes 1 and 2. Present the results as a polynomial numerator over polynomial denominator.
b) Give the conditions for this circuit to become a sinusoidal oscillator when there is a short circuit between nodes 1 and 2.

#. (6 points)



In the above circuit the op-amp has no current into its + & - leads and has the open circuit complex frequency gain (for which K and σ are positive numbers)

$$\frac{V_o}{V_d}(s) = \frac{K}{s+\sigma}$$

a) Give the circuit gain

$$\frac{V_o}{V_i}(s)$$

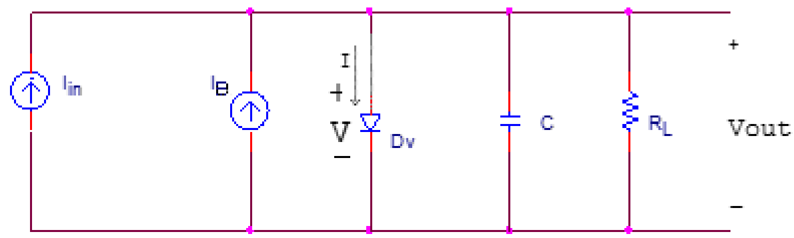
b) What type of filter is this?

#3. (7 points)

For the following diode circuit assume the diode, Dv, is described by the voltage V controlled by the current I via the cubic law:

$$V = V_d + K_d[I^3 - 10^{-6} I] \quad V_d = 1V,\ K_d = 6x10^8\ V/A^3$$

In the circuit C = 20nFd. I_B = 2mA is a bias current and I_{in} is a small signal current.



a) Sketch the diode V vs. I curve for –2mA< I <+2mA giving the local maxima and minima values of V.
b) Add a load line passing through the point V=0 at $I=I_B$, the bias current point of I_B =2mA, as well as the Q point, $V=V_Q$ at $I=I_Q=0$.
c) Find the value of the load resistance, R_L, to give this load line.
d) Find the small signal diode resistance r_d at the Q point.
e) Give the small signal differential equation for $V_{out}(t)$ with $I_{in}(t)$ as forcing function.
f) Give the small signal transfer function. $T(s) = \frac{V_{out}}{I_{in}}(s)$

Solutions – Circuits S2017

#1. a) $y_{in} = \dfrac{1}{R + \dfrac{1}{Cs + G_c + \dfrac{1}{Ls}}} = \dfrac{1}{R + \dfrac{Ls}{LCs^2 + LG_c s + 1}} = \dfrac{LCs^2 + LG_c s + 1}{RLCs^2 + RLG_c s + R + Ls}$

$(G_c = 1/R_c)$

$= \dfrac{LCs^2 + \frac{L}{R_c}s + 1}{RLCs^2 + \left(\frac{RL}{R_c} + L\right)s + R} = \dfrac{1}{R}\left[\dfrac{s^2 + \frac{1}{CR_c}s + \frac{1}{LC}}{s^2 + \frac{1}{C}\left(1 + \frac{R_c}{R}\right)s + \frac{1}{LC}}\right]$

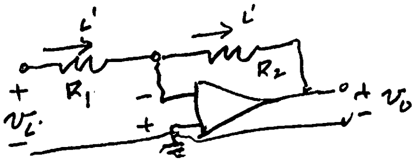
b) $I = y_{in}(s)V$

If node 1 is shorted to node 2, $V = 0 \Rightarrow I \neq 0$ if $y_{in}(s) \Rightarrow \infty$

$D(s) = RLCs^2 + L\left(\frac{R}{R_c} + 1\right)s + 1 = 0$; For sinusoidal $D(s) = as^2 + b$

$\Rightarrow RLC\left(s^2 + \frac{1}{RLC}\right) = 0$ with $\frac{1}{RLC} > 0 \Rightarrow \frac{R}{R_c} = -1 \Rightarrow \underline{R_c = -R < 0}$

#2.



$\Rightarrow v_i = R_1 i - v_d \Rightarrow i = G_1(v_i + v_d),\ G_1 = 1/R_1$

$-v_d = R_2 i + v_o \Rightarrow -v_d = R_2 G_1(v_i + v_d) + v_o$

$\Rightarrow -(1 + R_2G_1)v_d = R_2 G_1 v_i + v_o$

$\Rightarrow -(1 + R_2G_1)\dfrac{(s + \sigma_0)}{K}V_o = R_2 G_1 v_i + v_o \Rightarrow \left[-\dfrac{(1 + R_2G_1)}{K}(s + \sigma_0) - 1\right]v_o = R_2 G_1 v_i$

a) $\Rightarrow \dfrac{v_o}{v_i}(s) = \dfrac{-R_2 G_1 K}{[(1 + R_2G_1)(s + \sigma_0) + K]} = \dfrac{-[(R_2 G_1 K)/(1 + R_2 G_1)]}{s + \left(\sigma_0 + \frac{K}{1 + R_2 G_1}\right)}$

b) This is a low pass filter as $\frac{v_o}{v_i} \to 0$ monotonically as $s \to \infty$

#3. @ $I = \pm 10^{-3}$, $V = V_d = 1$, @ $I = -2\times10^{-3}$, $V = 1 - 6\times10^{8}(8\times10^{-9} - 2\times10^{-9}) = 1 - 3.6 = -2.6$

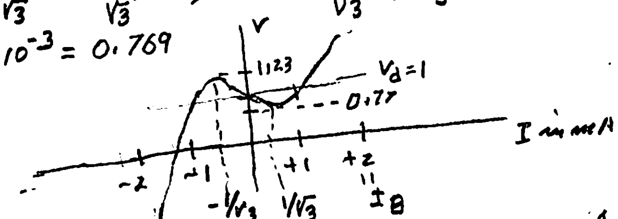
@ $I = +2\times10^{-3}$, $V = 1 + 6\times10^{8}(8\times10^{-9} - 2\times10^{-9}) = 1 + 3.6 = 4.6$

Max/Min $\Rightarrow dV/dI = K_d(3I^2 - 10^{-6}) = 0 \Rightarrow I = \pm\frac{1}{\sqrt{3}}$ mA $\Rightarrow$

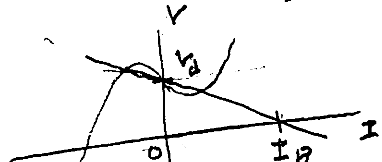
$V_{max} = V_d - K_d\left(\frac{1}{3\sqrt{3}}\times10^{-9} - \frac{1}{\sqrt{3}}\times10^{-9}\right) = 1 + \frac{6\times10^{-1}}{\sqrt{3}}\left(1 - \frac{1}{3}\right) = 1 + \frac{4}{\sqrt{3}}\times10^{-1} = 1.231$

$V_{min} = V_d - \frac{4}{\sqrt{3}}\times10^{-3} = 0.769$

a)



b)

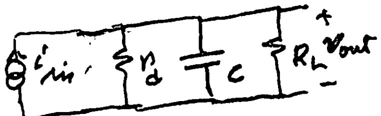


$R_L = -\text{slope} = -\left(\frac{V_d - 0}{0 - I_B}\right) = \frac{-1}{-2\times10^{-3}} = \frac{1000}{2} = 500\,\Omega$

c) $= -K_d\times10^{-6} = -6\times10^{8}\times10^{-6} = -600\,\Omega$

d) $r_d = \frac{dV}{dI} = K_d(3I^2 - 10^{-6})\Big|_{I_Q=0}$

e)



$i_{in} = [(g_d + G_L) + sC]v_{out} \Rightarrow C\frac{dv_{out}}{dt} + (g_d + G_L)v_{out} = i_{in}$

$g_d = 1/r_d = -1/600$, $G_L = 1/500$

$\Rightarrow 20\times10^{-9}\frac{dv_{out}}{dt} + (2 - 1.667)\times10^{-3}v_{out} = i_{in}$

f) $\dfrac{v_{out}}{i_{in}}(s) = \dfrac{1}{Cs + (g_d + G_L)} = \dfrac{1}{20\times10^{-9}s + 0.333\times10^{-3}} = \dfrac{(1/2)\times10^{8}}{s + 1.667\times10^{4}}$