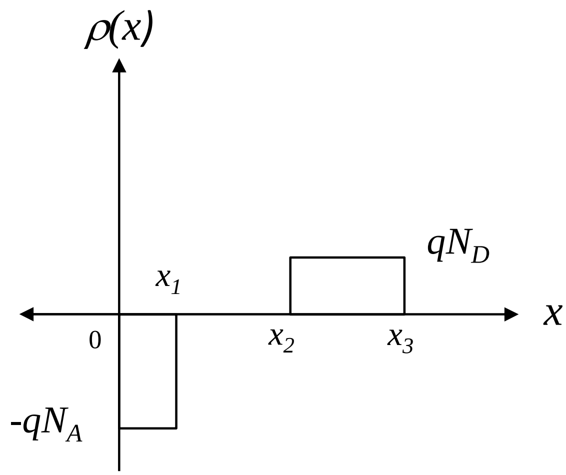
Devices

A helpful equation: $\frac{dE(x)}{dx} = \frac{\rho(x)}{\varepsilon_S}$



Problem #1 – (6 pts) Diode Basics

Consider a silicon diode with the charge density distribution illustrated on the right. Make LARGE labeled sketches of:

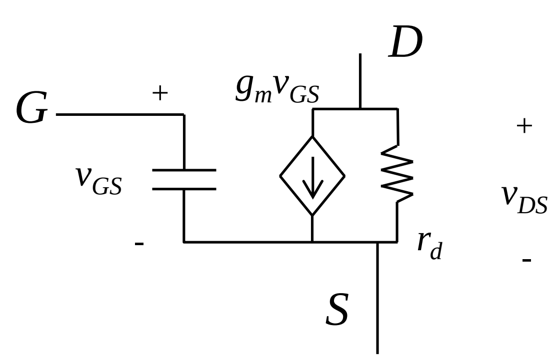
a) the resulting electric field *E(x)*. (Be sure to label the field at all relevant transitions.)
b) the potential $\phi(x)$. For this plot, assume that $\phi(0) = 0$.

Problem #2 – (7 pts) Diode Breakdown

In a reverse-biased pn-junction diode, there is a critical electric field, E_{crit}, at which breakdown occurs (whether due to a tunneling or an avalanche mechanism). Given the depletion region width equation $W_{depletion} = \sqrt{\frac{2\varepsilon_s}{q}\left(\frac{1}{N_A} + \frac{1}{N_D}\right)V_0}$ solve for the applied reverse bias voltage V_R where breakdown occurs. V_0 represents the 'built-in voltage', N_A and N_D represent the doping concentrations in *p* and *n* regions, and ε_S represents the permittivity of silicon.

Problem #3 – (7 pts) n-channel Enhancement MOSFET in triode

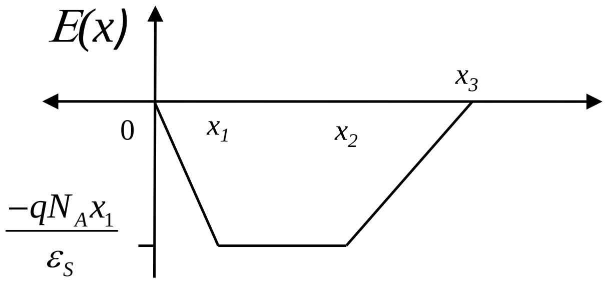
Occasionally it becomes important to model the small signal operation of a MOSFET in the triode (aka 'ohmic') mode of operation. Using the triode equation given below, solve for the transconductance (g_m) and drain resistance (r_d) that can be used to model the transistor at a fixed operating point where $v_{GS} - V_{th} > v_{DS} > 0$. The two parameters g_m and r_d capture the effect of the gate and drain voltages on the drain current.



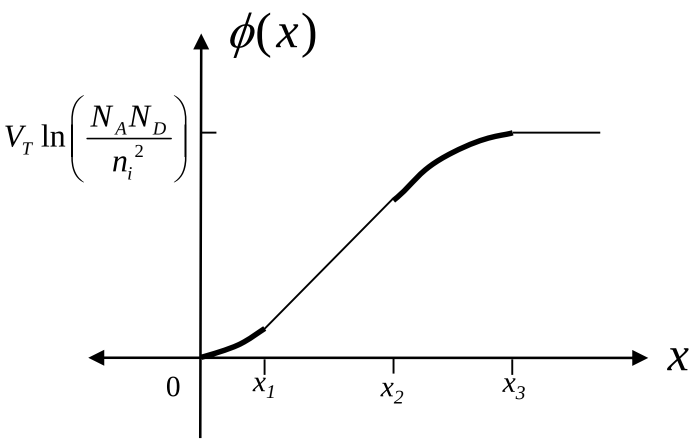
$i_D = k_n' \frac{W}{L}\left[\left(v_{GS} - V_{th}\right)v_{DS} - \frac{1}{2}v_{DS}^2\right]$ where k_n', *W*, *L*, and V_{th} are the constant parameters: process transconductance, channel width, channel length, and threshold voltage, respectively.

Problem #1

The slope of the electric field is proportional to the charge density and so we have a linearly decreasing electric field where the charge density is fixed and negative ($0 < x < x_1$) , followed by a zero slope in the electric field, and terminated by a linearly rising region where the charge density is fixed and positive ($x_2 < x < x_3$). In a diode, the depletion charge balance each other and so the electric field well away from the junction is zero.



The linearly decreasing and increasing regions of the electric field ($0 < x < x_1$ and $x_2 < x < x_3$) produce quadratically-shaped regions in the potential plot, whereas the constant electric field region ($x_1 < x < x_2$) produces a linear region in the potential.



Problem #2 – Diode Breakdown

In a reverse-biased pn-junction diode, there is a critical electric field, E_{crit}, at which breakdown occurs, whether due to a tunneling or an avalanche mechanism. Given the depletion region width equation $W_{depletion} = \sqrt{\frac{2\varepsilon_s}{q}\left(\frac{1}{N_A} + \frac{1}{N_D}\right)V_0}$ (where V_0 represents the 'built-in voltage'), solve for the applied reverse bias voltage V_R where breakdown occurs.

To solve this problem, there are several things that must be known: 1) $x_n N_D = x_p N_A$ due to charge balance, 2) that the peak electric field occurs at the junction and is:

$|E_{peak}| = x_p \frac{qN_A}{\varepsilon_S} = x_n \frac{qN_D}{\varepsilon_S}$ (where x_p and x_n represent the depletion region widths on each side of the junction), and 3) that the reverse-bias voltage V_R is added to the V_0 in the width equation.

$x_n N_D = x_p N_A$ gives us: $x_p = x_n \frac{N_D}{N_A}$, so $W_{depletion} = x_n + x_p = x_n + x_n \frac{N_D}{N_A} = x_n \left(\frac{N_A + N_D}{N_A} \right)$

$$x_n \left(\frac{N_A + N_D}{N_A} \right) = \sqrt{\frac{2\varepsilon_s}{q} \left(\frac{1}{N_A} + \frac{1}{N_D} \right)(V_0 + V_R)} \quad \Rightarrow \quad x_n = \sqrt{\frac{2\varepsilon_s}{q} \left(\frac{N_A}{(N_D + N_A) N_D} \right)(V_0 + V_R)}$$

To find what V_R produces an $|E_{peak}| = E_{crit}$, we solve for V_R where,

$$E_{crit} = x_n \frac{qN_D}{\varepsilon_S} = qN_D \sqrt{\frac{2}{q\varepsilon_s} \left(\frac{N_A}{(N_D + N_A) N_D} \right)(V_0 + V_R)}$$

$$\boxed{\frac{E_{crit}{}^2 \varepsilon_s}{2q} \frac{N_D + N_A}{N_D N_A} - V_0 = V_R}$$

Problem #3

The dependent current source represents how the gate voltage affects the drain current $(\partial i_D / \partial v_{GS})$ and the drain resistance represents how the drain voltage affects the drain current $(\partial i_D / \partial v_{DS})$.

$$i_D = k_n' \frac{W}{L} \left[(v_{GS} - V_t) v_{DS} - \frac{1}{2} v_{DS}{}^2 \right]$$

$$\boxed{g_m = \frac{\partial i_D}{\partial v_{GS}} = k_n' \frac{W}{L} V_{DS}} \quad \text{and} \quad g_d = \frac{\partial i_D}{\partial v_{DS}} = k_n' \frac{W}{L} \left[(V_{GS} - V_t) - V_{DS} \right] \quad \text{so,}$$

$$\boxed{r_d = \frac{1}{g_d} = \frac{1}{k_n' \frac{W}{L} \left[(V_{GS} - V_t) - V_{DS} \right]}}$$