Solutions

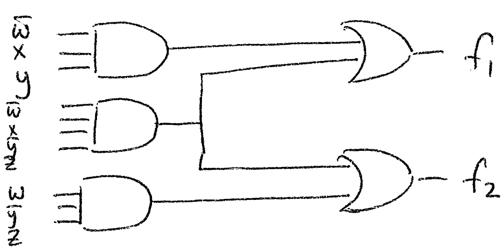
ECE Written Qualifying Examination, Winter 2017
Digital Logic

1. (4 points) Boolean Simplification.

   Give an example of a pair of Boolean functions $(f_1(w,x,y,z), f_2(w,x,y,z))$ such that the minimal two-output network for f_1, f_2 contains a product term that is not a prime implicant of f_1 and not a prime implicant of f_2. Justify your answer and draw the minimal two-output network.

	00	01	11	10	yz
00					
01		1	1	1	f_1
x 11		1			
10		1			
		f_2			

		cost:	
minimize f_1:	$\bar{w}xz + \bar{w}xy$	8	= 16
minimize f_2:	$x\bar{y}z + w\bar{y}z$	8	
combined f_1, f_2:	$\bar{w}x\bar{y}z + \bar{w}xy$		= 14
	$\bar{w}x\bar{y}z + w\bar{y}z$		



2. (4 points) Boolean Algebra.

   Using Boolean Algebra postulates and theorems Prove that

$$\overline{x}\,\overline{z} + x\overline{y} = \overline{y}\,\overline{z} + \overline{x}\,\overline{z} + x\overline{y}.$$

   No credit will be given for solutions that use the truth table method.

   Hint: Start by multiplying one of the product terms on the right hand side by 1.

$RHS = \bar{y}\bar{z} + \bar{x}\bar{z} + x\bar{y}$	
$= \bar{y}\bar{z}\cdot 1 + \bar{x}\bar{z} + x\bar{y}$	multiplicative identity
$= \bar{y}\bar{z}(x+\bar{x}) + \bar{x}\bar{z} + x\bar{y}$	complement
$= \bar{y}\bar{z}x + \bar{y}\bar{z}\bar{x} + \bar{x}\bar{z} + x\bar{y}$	distributive
$= \bar{x}\bar{z} + \bar{x}\bar{z}\bar{y} + x\bar{y} + x\bar{y}\bar{z}$	commutative
$= \bar{x}\bar{z} + x\bar{y}$	absorption
$= LHS$	

3. (6 points) The following figure gives the circuit diagram of a mod-10 counter that counts the sequence 0000, 0001, 0010, . . ., 1001, and then repeats from 0000.

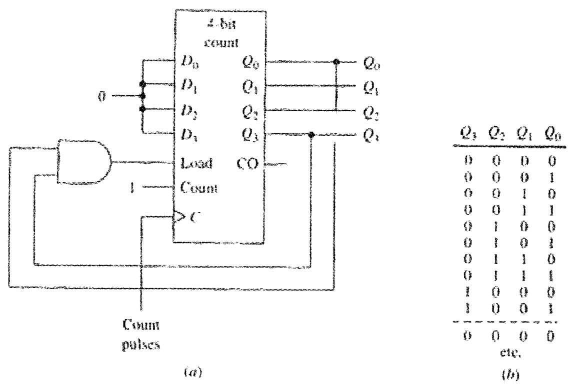
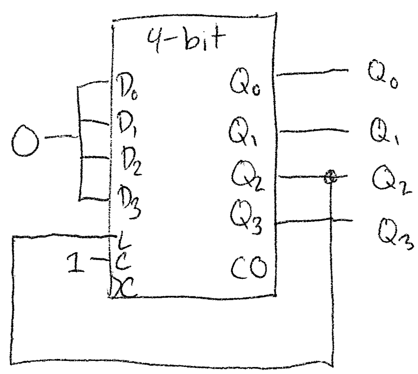
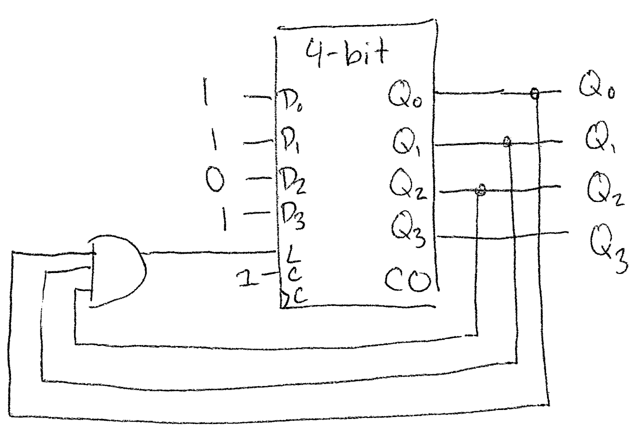


Figure 6.35 Synchronous mod-10 counter. (a) Connections. (b) Counting sequence.

(a) (3 points) Draw a similar circuit diagram for a mod-5 counter that counts the sequence 0000, 0001, 0010, 0011, 0100, and then repeats from 0000.



(b) (3 points) Draw a similar circuit diagram for a mod-5 counter that counts the sequence1011, 1100, 1101, 1110, 1111, and then repeats from 1011.



4. (6 points) State Diagram. You are given the state diagrams of two single-input, single-output Mealy-model finite state machines (FSMs) A and B, both of which take the same input x. FSMs A and B have n_A and n_B number of states, respectively. Explain how you will use the state diagrams of A and B to draw the state diagram of a Mealy-model FSM C that takes x as input and outputs a 1 whenever A or B would output a 1 (and outputs a 0 otherwise). How many states will C have? How will you determine the transitions in C, given the transitions in A and B?

Let $\mathcal{S}_A$ be the states of A

$\mathcal{S}_B$ be the states of B

Set of states of C: $\mathcal{S}_C := \{(S_i, S_j) : S_i \in \mathcal{S}_A, S_j \in \mathcal{S}_B\}$

number of states: $n_A \cdot n_B$

initial state of C is (S_{Ainit}, S_{Binit}) where S_{Ainit}, S_{Binit} are initial states of A, B

for any pair of states $(S_i, S_j), (S_k, S_l) \in \mathcal{S}_C$

there is an edge labeled $1/0$ $(0/0)$ from (S_i, S_j) to (S_k, S_l)

iff: - There is an edge labeled $1/0$ $(0/0)$ from S_i to S_k in A AND

- There is an edge labeled $1/0$ $(0/0)$ from S_j to S_l in B

there is an edge labeled $1/1$ $(0/1)$ from (S_i, S_j) to (S_k, S_l)

iff: - There is no $1/0$ $(0/0)$ edge as above AND

- There is an edge labeled $1/*$ $(0/*)$ from S_i to S_k in A AND

- There is a edge labeled $1/*$ $(0/*)$ from S_j to S_l in B.