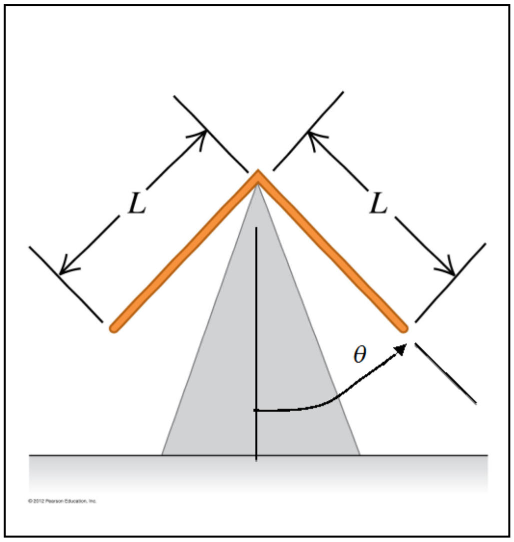
Basic Physics

1. (7 pts) A pendulum is formed by two uniform mass density rods of length *L*, mass *m*, joined at right angles and placed on a knife edge so that they can pivot. One of the rods makes an angle θ with respect to the vertical (gravitational acceleration constant g)



(A) What is the potential energy of the configuration as a function of the given variables and parameters. What is the equilibrium value of the angle. (3ps)
(B) When the pendulum pivots the angle becomes a function of time, $\theta(t)$. What is the kinetic energy as a function of the given variables and parameters and $d\theta / dt$. (2pts)
(C) Using your answers to (A) and (B) what is the frequency of oscillations when the excursions are small? (2 pts)

Solution:

A. Gravitational potential energy

$$V = mg(\Delta h_1 + \Delta h_2)$$

$$\Delta h_1 = (L/2)(1 - \cos\theta)$$

$$\Delta h_2 = (L/2)(-\sin\theta)$$

Equilibrium

$$\frac{\partial V}{\partial \theta} = 0 = (mgL/2)(\sin\theta - \cos\theta)$$

$$\theta_E = \pi / 4$$

B. Kinetic energy

$$T = 2 \cdot \frac{1}{2}\int_0^L dl \frac{dm}{dl}\left(l\dot{\theta}\right)^2$$

$$dm/dl = m/L$$

$$T = \frac{m}{3}L^2\dot{\theta}^2$$

Small Oscillations will be sinusoidal functions of time

For Harmonic oscillation $\delta\theta(t) = \delta \sin \omega t$.

Total energy

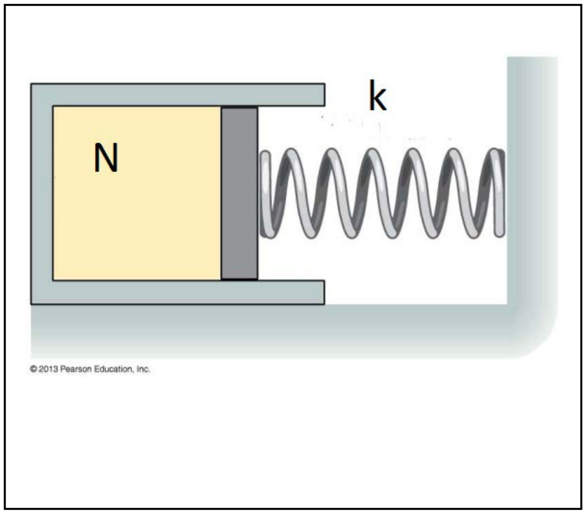
$$T + V = \frac{m}{3} L^2 \left(\delta\dot{\theta}\right)^2 + \frac{mgL}{2\sqrt{2}} \delta\theta^2$$

$$T + V = \frac{m}{3} L^2 \left(\omega\right)^2 \delta^2 \cos^2 \omega t + \frac{mgL}{2\sqrt{2}} \delta^2 \sin^2 \omega t$$

For energy to be constant

$$\omega = \left(\frac{3}{2\sqrt{2}}\right)^{1/2} \sqrt{\frac{g}{L}}$$

2. (7 pts) An ideal monatomic gas of *N* atoms is held in a cylinder by a piston with area *A* attached to a compressed spring with Hooke's constant *k*. The initial absolute temperature is *T* and Boltzmann's constant is k_B. (A monatomic molecule has three dergees of freedom. Hence, in thermal equilibrium each molecule has $\frac{3}{2}k_BT$ of energy.)



(A) If the spring is compressed by an amount *x*, then in terms of the given variables and parameters,what is the volume of the gas? (2 pts)

(B) A small amount of heat energy *dQ* is given to the gas. The walls of the cylinder are maintained at temperature *T*, what is the change in spring compression *dx*? (2 pts)

(C) Now suppose the walls of the cylinder are thermally insulating, what is the change in spring compression, dx, in response to dQ?. (3 pts)

Solution

For an ideal monatomic gas the internal energy U is given by $U = \frac{3}{2}Nk_BT$, and the pressure satisfies $PV = Nk_BT$.

(A) The force on the piston from the spring is $F = kx$. This is balanced by the pressure force $F = PA$. So $V = Nk_BTA / kx$.

(B) If the temperature of the walls is maintained, all the added heat energy flows to the walls and there is no change in spring compression, dx=0.

(C) If the walls are thermally insulating the internal energy increases by the amount oof added heat less the work done against the spring in expanding,

$$dU = dQ - PdV.$$

Rewriting the energy in terms of the pressure and volume, $U = \frac{3}{2}PV$.

$$dU = \frac{3}{2}\left(PdV + VdP\right) = dQ - PdV$$

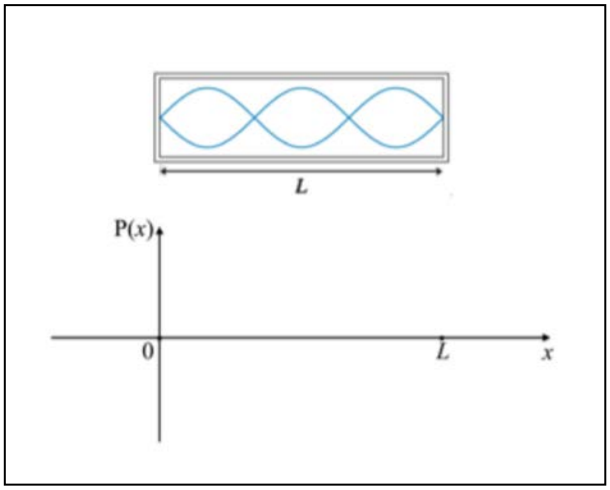
The change in volume is $dV = Adx$. The change in pressure is $dP = kdx / A$.

Thus,

$$dQ = \frac{5}{2}PdV + \frac{3}{2}VdP = \left[\frac{5}{2}PA + \frac{3}{2}V\frac{k}{A}\right]dx$$

$$dx = \left[\frac{5}{2}PA + \frac{3}{2}V\frac{k}{A}\right]^{-1} dQ$$

3. (6 pts) The figure shows a wave function with quantum number *n* for an electron confined in a one-dimensional box (Length *L*).



(a) Given the wave function as shown, what is the corresponding quantum number *n*?

(b) Now the electron makes a transition from this excited state, *n*, to the ground *n*=1 state, emitting a photon of wavelength λ. What is the length *L* of the box? (give your answer in terms of *m*, λ,and $\hbar$). If the length of the box is doubled and the electron makes the same transition between these two quantum states (i.e., from the excited state, *n*, to the ground *n*=1 state), will the emission wavelength decrease, increase or be the same?

(c) Given the wave function as shown for the excited state, draw schematically its probability density along *x*. What is the probability of finding the electron somewhere inside the box?

Solution:

(a) n=3 ,

(b) The electron energy states are $E = \frac{(\hbar k)^2}{2m}$, $k = \frac{n\pi}{L}$

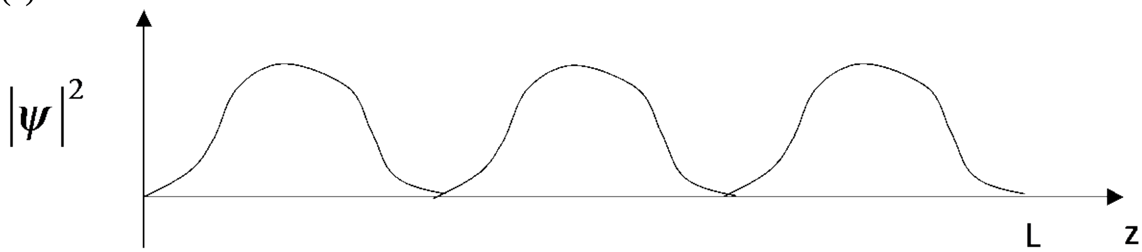
Transition

$$E_{3\to 1} = (3^2 - 1^2)\frac{(\hbar\pi / L)^2}{2m} = hc / \lambda$$

$$L^2 = 8\frac{(\hbar\pi)^2}{2m}\frac{\lambda}{hc} = \frac{h\lambda}{mc}$$

If L is doubled wavelength increases by a factor of 4.

(c)



Probability =1, Max $|\psi|^2 = 2/L$ above