Qual exam ECE/ Basic Physics: Fall 2018

1. Sliding blocks (7 pts.)

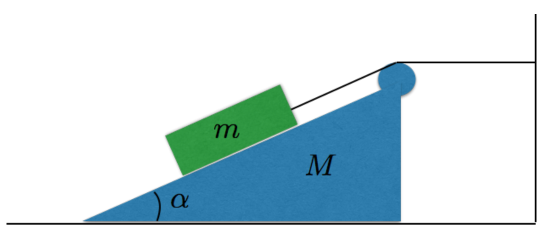
A box of mass m presses on another wedge of mass M and angle α. The top box is attached to a rope, which is fixed to a vertical wall. We ignore friction in this problem. The gravitational acceleration is g. Find the acceleration of mass M. Any approach is accepted. You can also follow these steps:

(a) Show all forces on both blocks, i.e., the force diagram. a_M is the acceleration of M with respect to the bottom surface and a_0 is the acceleration of m with respect to the wedge and a_m is the acceleration of m in the rest frame. Indicate a_M, a_m and a_0 on the diagram. (1 pt.)

(b) Given the geometrical constraints, find one relation between a_M, a_m and a_0. It doesn't have to involve all of them. (2 pts.)

(c) Using the force diagram from part (a) find the relation between acceleration and force, i.e., equations of motion. (2 pts.)

(d) Find a_M. (2 pts.)

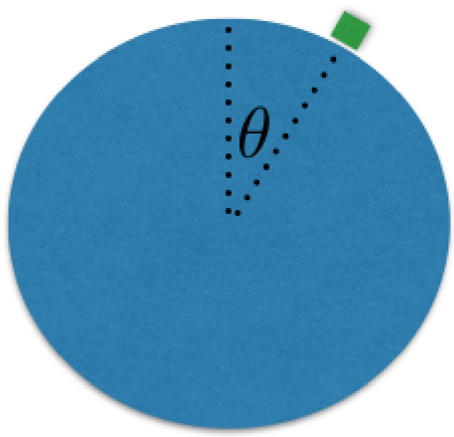


2. Sliding on a sphere (6 pts.)

A body of mass m starts off sliding from a the top of a smooth sphere of radius R. The gravitational acceleration is g. The initial velocity is assumed to be very small. The goal is to find the angle θ as which the body leaves the surface of the sphere. Optional: you can follow these steps:

(a) Using the conservation of energy find the velocity of the body, for a given angle. (3 pts.)

(b) Using Newton's second law, find the normal force on the surface, and consequently θ. (3 pts.)



3. Spin-1/2 measurement (7 pts.)

We denote spin-1/2 states along the ±z-directions by $|\uparrow\rangle$ and $|\downarrow\rangle$, respectively.
(a) We start with the state $|\uparrow\rangle$. We measure the spin in the z-direction, corresponding to S_z operator. What are outcomes and their respective probabilities? (1 pts.)

(b) Then, we measure the spin in the x-direction, corresponding to S_x operator. What are outcomes and their respective probabilities? (2 pts.)

(c) Then, we again measure the spin in the z-direction. What are outcomes and their respective probabilities? (2 pts.)

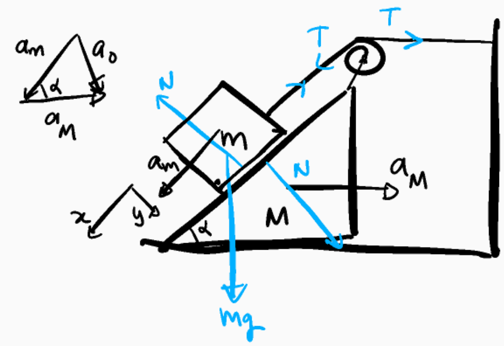
(d) Now, consider two spin-1/2 particles (A and B) prepared in the singlet state:

$$|S\rangle = \frac{1}{\sqrt{2}}(|\uparrow\rangle_A|\downarrow\rangle_B - |\downarrow\rangle_A|\uparrow\rangle_B)$$

We measure spin A in the z-direction and get $-\hbar/2$. Then, we measure spin B in the z-direction. What are outcomes and their respective probabilities? (2 pts.)

$a_m = a_M = a'$

since the same rope dictates the displacement.



$a'^2 + a'^2 - 2\cos\alpha\, a'^2 = a_0^2$

$$\Rightarrow \quad a_0 = a'\sqrt{2(1-\cos\alpha)}$$

from the rope

For mass M, in the horizontal direction : $N\sin\alpha + T(1-\cos\alpha) = Ma'$

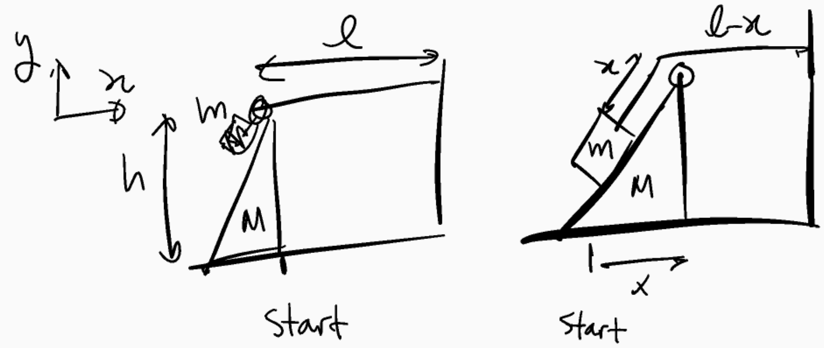
For mass m, in x-direction : $mg\sin\alpha - T = m(a' - a'\cos\alpha)$

For mass m, in y-direction : $+mg\cos\alpha - N = ma'\sin\alpha$

$$\Rightarrow \quad a' = \frac{mg\sin\alpha}{M + 2m(1-\cos\alpha)}$$

① Alternative method: conservation of energy

$$mg\,x\sin\theta = \frac{1}{2} M \dot{x}^2 + \frac{1}{2} m v^2$$



position of m: if zero at start $(x - x\cos\theta,\ -x\sin\theta)$

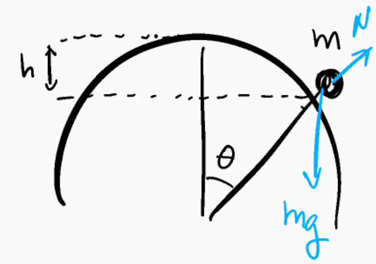
$$\Rightarrow v^2 = \dot{x}^2\left((1-\cos\theta)^2 + \sin^2\theta\right) = 2\dot{x}^2\left[1-\cos\theta\right]$$

$$mg\,x\sin\theta = \frac{1}{2}\left[M + 2m(1-\cos\theta)\right]\dot{x}^2$$

$$mg\,\dot{x}\sin\theta = \frac{1}{\cancel{2}}\left[M+2m(1-\cos\theta)\right]\cancel{2}\,\dot{x}\,\ddot{x}$$

$$\Rightarrow \ddot{x} = \frac{mg\sin\theta}{M+2m(1-\cos\theta)}$$

②



$$mgh = \frac{1}{2}mv^2$$

$$h = R(1-\cos\theta)$$

$$\rightarrow v^2 = 2gR(1-\cos\theta)$$

$mg\cos\theta - N = mv^2/R$, for departure $N=0$

$$\rightarrow mg\cos\theta = 2gm(1-\cos\theta)$$

$$\rightarrow \cos\theta = 2/3 \;,\; v = \sqrt{2gR/3}$$

(a) $\hbar/2$ with 100% probability.

(b) $\begin{cases} +\hbar/2 & \text{with 50\% probability} \\ -\hbar/2 & \text{" \quad "} \end{cases}$

(c) $\begin{cases} +\hbar/2 & \text{with 50\% probability} \\ -\hbar/2 & \text{" \quad "} \end{cases}$

regardless of the outcome in (b)

(d) $\hbar/2$ for spin B with 100% probability