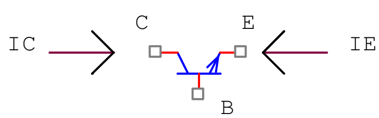
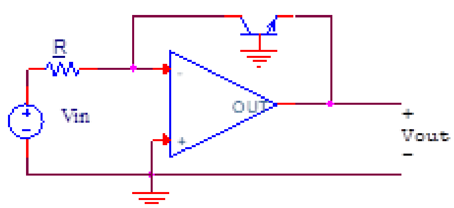
Circuits Fall 2018

● For the following circuits, unless otherwise stated, assume that the op-amp is ideal (zero input currents and zero input difference voltage [due to infinite gain]).
● And assume that a npn transistor is described by the Ebers-Moll model which has for the collector current:

$$I_C=\alpha I_{Eo}\,(1-\exp(V_{BE}/V_T)) - I_{Co}(1-\exp(V_{BC}/V_T))$$

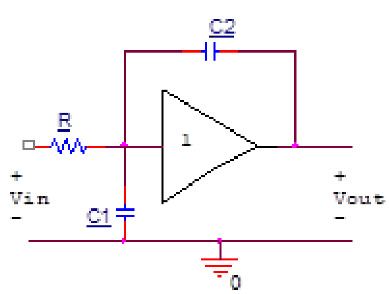


#1. (6 points) For the following feedback circuit:



a) (4 points) Find Vout as a function of Vin .
b) (2 points) If the op-amp is not ideal and actually saturates at supply bias voltages $\pm V_{supply}$, discuss limitations on the validity of your answer in part a)

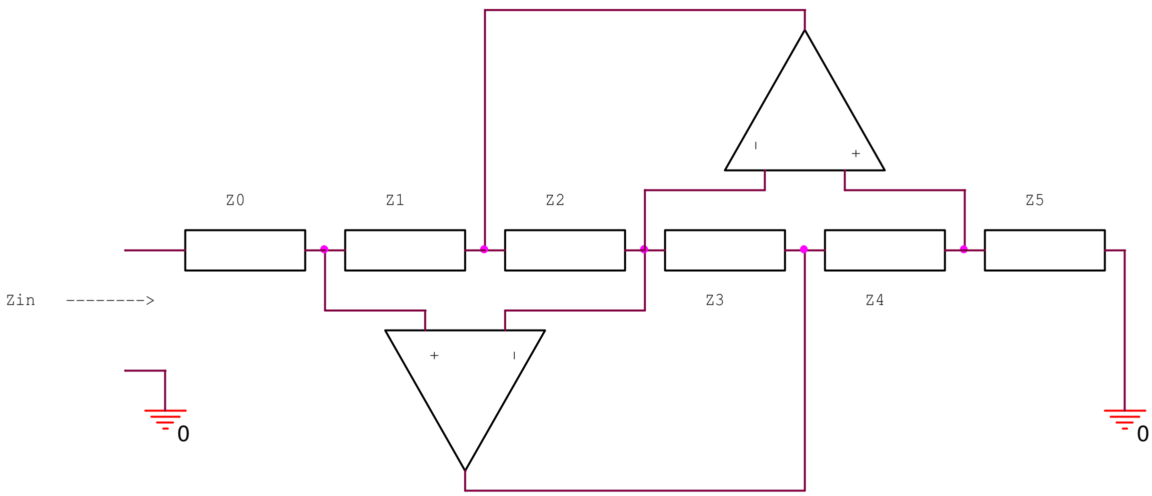
#2. (7 points) In the following circuit the amplifier is a unity voltage gain one with no input current



(a) (3points) Find the transfer function Vout(s)/Vin(s)
(b) (2 points) Give the zeros and poles of the transfer function and the unit impulse response.
(c) (2 points) Give an ideal op-amp realization of the amplifier.

#3) (7 points) The following is a circuit diagram for a General Impedance Converter (GIC) for which the input impedance is given as:

$$Zin(s) = Z0(s)+(Z1(s)\cdot Z3(s)\cdot Z5(s))/(Z2(s)\cdot Z4(s))$$

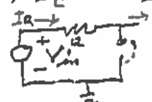


Choose Z2 and Z4 as equal capacitors of capacitance C and all other branches as resistors with equal resistance R.

a) (2 points) Give Zin(s) in terms of the chosen components.
b) (3 points) Give the poles and zeros of Zin(s).
c) (2 points) The circuit can be an oscillator. Give conditions for this and the oscillation frequency in terms of R and C.

Circuits F2018 Solutions

#1. a) as $v_{BC} = v_{op\text{-}amp\text{-}in} = 0$ then $I_C = \alpha I_{E_0}(1 - e^{V_{BE}/V_T}) = \alpha I_{E_0}(1 - e^{-V_{out}/V_T})$ as $V_{BE} = -V_{out}$

and as $v_{op\text{-}amp\text{-}in} = 0 \Rightarrow$  $I_R = I_C$; $I_R = V_{in}/R = I_C = \alpha I_{E_0}(1 - e^{-V_{out}/V_T})$

$$\Rightarrow \frac{V_{in}}{\alpha I_{E_0} R} - 1 = -e^{-V_{out}/V_T}$$

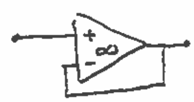
$$\Rightarrow V_T \ln\left(1 - \frac{V_{in}}{\alpha I_{E_0} R}\right) = -V_{out} \Rightarrow V_{out} = -V_T \ln\left(1 - \frac{V_{in}}{\alpha I_{E_0} R}\right)$$

b) as $-12 < V_{out} < +12$ outside of saturation $-V < -V_T \ln\left(1 - \frac{V_{in}}{\alpha I_{E_0} R}\right) < +V$, $V = |V_{supply}|$, limits V_{in} to cover V_{out} in the $\ln(.)$ region.

#2. a) as the amplifier output voltage equals its input voltage the voltage on C_2 is zero & no current flows into C_2. Then RC_1 acts as a voltage divider  $v_C(s) = \frac{1/sC}{R + 1/sC} V_{in} = \frac{1}{RCs+1} V_{in} = V_{out}$ by unity gain amplifier

b) $T(s) = \frac{1}{RCs+1} = \frac{1/RC}{s + 1/RC}$ = transfer function $\frac{V_{out}}{V_{in}}$; impulse response $h(t) = \frac{1}{RC} e^{-t/RC}\, 1(t)$ ← unit step

$\Rightarrow$ zero @ $s = \infty$, pole @ $s + 1/RC = 0 \Rightarrow$ @ $s = -1/RC$

c)  ≡  as $V_+ = V_-$ & $I_+ = 0 \Rightarrow V_{out} = V_{in}$ & $I_{in} = 0$

#3. a) $Z_2(s) = Z_4(s) = \frac{1}{Cs}$, $Z_0 = Z_1 = Z_2 = Z_3 = R \Rightarrow Z_{in}(s) = R + R^3 \cdot \frac{1}{\left(\frac{1}{Cs}\right)^2} = R + R^3 C^2 s^2$

b) poles, two @ $s = \infty$, zeros $\Rightarrow 0 = R + R^3 C^2 s_0^2 \Rightarrow s_0^2 = -\frac{R}{R^3 C^2} = -\frac{1}{(RC)^2} \Rightarrow s_{0\pm} = \pm j \frac{1}{RC}$

as $V_{in}(s) = (R + R^3 C^2 s^2) I_{in}$

short circuit $\Rightarrow V_{in} = 0 \Rightarrow I_{in} \neq 0$ if $Z_{in}(s) = 0 \Rightarrow s_{0\pm}$ are short circuit natural frequencies

open circuit $\Rightarrow I_{in} = 0 \Rightarrow V_{in} \neq 0$ if $Z_{in}(s) = \infty \Rightarrow s = \infty$'s are open circuit natural frequencies

c) To be an oscillator initial conditions to excite the circuit to give sine waves. This occurs at the natural frequencies & for finite ones when the input is shorted and

$s_0 = j\omega_0 \Rightarrow \omega_0 = \frac{1}{RC} \equiv f_{osc} = \frac{1}{2\pi RC}$ Hertz