SOLUTIONS

Problem 1 – Carrier Diffusion in 1-D

Consider the one-dimensional problem of an extremely narrow, uniformly-doped, p-type bar of silicon of length L with an ohmic contact on the right-hand side at x = L. Electron-hole pairs are being generated at a *rate* of G .

The minority carrier diffusion length constant Le is neither very-short nor very-long compared to L. Assume that you know the minority carrier diffusion constant D_n. The homogeneous solution is: $n'(\mathrm{x}) = \mathrm{A}e^{\frac{x}{Le}} + Be^{\frac{-x}{Le}}$

a) (5 pts) Find an expression for the excess minority carrier concentration *n'(x)* from x = 0 to x = L.
b) (2 pts) Unlike in part (a), if we could assume that $L >> Le$, find an expression for the excess minority carrier concentration *n'(x)* from x = 0 to x = L.

1a) $n'(x) = Ae^{\frac{x}{Le}} + Be^{\frac{-x}{Le}}$ with the boundary conditions: $n'(\mathrm{L}) = 0$ and $Flux(x=0) = G = -D_n \dfrac{dn'(0)}{dx}$

Using the first boundary condition: $n'(L) = 0 = Ae^{\frac{L}{Le}} + Be^{\frac{-L}{Le}}$ ➔ $-Ae^{\frac{L}{Le}} = Be^{\frac{-L}{Le}}$ ➔ $\boxed{A = -Be^{\frac{-2L}{Le}}}$

Using the second boundary condition: $G = -D_n \dfrac{dn'(0)}{dx} = -D_n\left(\dfrac{A}{Le}e^0 - \dfrac{B}{Le}e^0\right) = \dfrac{-D_n}{Le}(A-B)$

$$G = \frac{-D_n}{Le}(A-B) = \frac{-D_n}{Le}\left(-Be^{\frac{-2L}{Le}} - B\right) = B\frac{D_n}{Le}\left(e^{\frac{-2L}{Le}} + 1\right) \text{ ➔ } \boxed{B = \frac{G\cdot Le}{D_n\left(e^{\frac{-2L}{Le}}+1\right)}}$$

and plugging in,

$$\boxed{A = -\left(\frac{G\cdot Le}{D_n\left(e^{\frac{-2L}{Le}}+1\right)}\right)e^{\frac{-2L}{Le}} = \frac{-G\cdot Le}{D_n}\frac{e^{\frac{-2L}{Le}}}{e^{\frac{-2L}{Le}}+1}}$$

so we get:

$$n'(x) = \frac{-G\cdot Le\cdot e^{\frac{-2L}{Le}}}{D_n\left(e^{\frac{-2L}{Le}}+1\right)}\cdot e^{\frac{x}{Le}} + \frac{G\cdot Le}{D_n\left(e^{\frac{-2L}{Le}}+1\right)}\cdot e^{\frac{-x}{Le}}$$

1b) $L >> Le$

$n'(x) = Ae^{\frac{x}{Le}} + Be^{\frac{-x}{Le}}$ with the boundary conditions: $n'(\mathrm{L}) = 0$ and $Flux(x=0) = G = -D_n \dfrac{dn'(0)}{dx}$

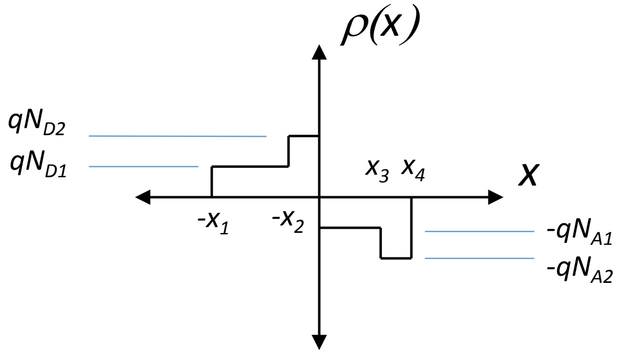
Using the first boundary condition: $n'(L) = 0 = Ae^{\frac{L}{Le}} + Be^{\frac{-L}{Le}}$ ➔ $-Ae^{\frac{L}{Le}} = Be^{\frac{-L}{Le}}$ ➔ $A = -Be^{\frac{-2L}{Le}}$ but because $L >> Le$, $A \approx 0$ and the solution takes the simple decaying exponential form: $n'(x) \approx Be^{\frac{-x}{Le}}$

Using the second boundary condition: $G = -D_n \dfrac{dn'(0)}{dx} = -D_n\left(-\dfrac{B}{Le}e^0\right) = D_n\dfrac{B}{Le}$ ➔ $\dfrac{G\cdot Le}{D_n} = B$ Therefore,

$$n'(x) \approx \frac{G\cdot Le}{D_n}e^{\frac{-x}{Le}}$$

Problem 2 – PN-junction diode

Consider a diode with two-level doping on each side. Use the resulting depletion region charge distribution shown in the figure on the right.



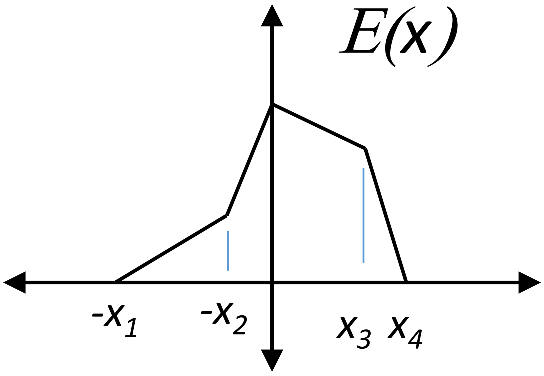
a) (2 pts) If you are given –x1, -x2, and x4, find an expression for x3. Be sure to show your steps and explain any assumptions you use.

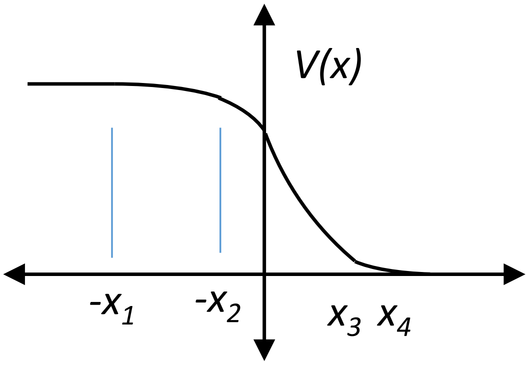
b) (5 pts) Make a BIG, unambiguous sketch of both the resulting electric field (vs x) and the potential (vs x). Be sure to clearly label where x1, x2, x3, and x4 are. You do NOT need to label the electric field or potential levels.

$$(x_1 - x_2)N_{D1} + x_2 N_{D2} = (x_4 - x_3)N_{A2} + x_3 N_{A1} \quad \text{solving for } x_3$$

$$(x_1 - x_2)N_{D1} + x_2 N_{D2} - x_4 N_{A2} = x_3\left(N_{A1} - N_{A2}\right)$$

$$\boxed{\frac{(x_1 - x_2)N_{D1} + x_2 N_{D2} - x_4 N_{A2}}{N_{A1} - N_{A2}} = x_3}$$





Problem 3 – N-Channel Enhancement-Mode MOSFET

The standard formula for the above-threshold, saturation-mode, drain current in the nFET is: $I_D = \frac{1}{2}k_n{}' \frac{W}{L}\left(V_{GS} - V_t\right)^2$

where V_{GS} is the gate to source voltage, V_t is the threshold voltage, $k_n{}'$ is the process transconductance, and W/L is the width to length ratio. In the triode mode of operation, the drain current equation is given as:

$$I_D = k_n{}' \frac{W}{L}\left[\left(V_{GS} - V_t\right)V_{DS} - \frac{1}{2}V_{DS}{}^2\right]$$

a) (3 pts) If the source is tied to ground (0 volts) and the gate and drain are connected together, we have the "diode configuration". Assuming saturated, above-threshold operation, solve for the small signal equivalent resistance seen to ground as a function of V_{GS}.

b) (3 pts) In triode, we can also obtain a small signal equivalent resistance by biasing the gate above threshold, and operating the drain voltage separately. Find the expression for this resistance from the drain to ground as a function of V_{GS}.

3a) The small signal equivalent conductance would be: $\dfrac{dI_D}{dV_{GS}} = d\left(\dfrac{1}{2}k_n{}'\dfrac{W}{L}\left(V_{GS} - V_t\right)^2\right) / dV_{GS} = k_n{}'\dfrac{W}{L}\left(V_{GS} - V_t\right)$

and the equivalent resistance would then be: $\boxed{r_{eq} = \dfrac{1}{k_n{}'\dfrac{W}{L}\left(V_{GS} - V_t\right)}}$

3b) $g_D = \dfrac{dI_D}{dV_D} = k_n{}'\dfrac{W}{L}\left[\left(V_{GS} - V_t\right) - V_{DS}\right]$

$$\boxed{r_D = \frac{1}{k_n{}'\dfrac{W}{L}\left[\left(V_{GS} - V_t\right) - V_{DS}\right]}}$$