Solutions

ECE Written Qualifying Examination, Fall 2018
Digital Logic

1. (5 points) Boolean Simplification.

   Using the Quine-McCluskey method, obtain all the prime implicants for

$$f(x, y, z) = \sum m(0, 2, 3, 5, 7).$$

(0) 000	(0,2) 0-0
(2) 010	(2,3) 01-
(3) 011	(3,7) -11
(5) 101	(5,7) 1-1
(7) 111	

2. (5 points) Boolean Algebra.

   Using Boolean Algebra postulates and theorems prove that

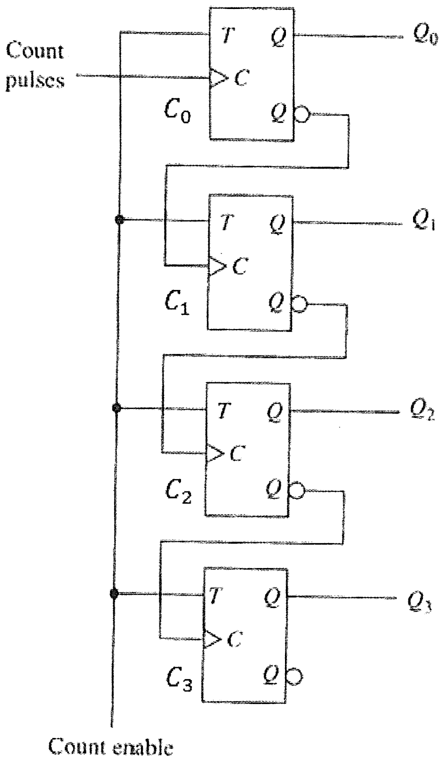
$$(x + z)(y + z)(\overline{y} + \overline{z}) = \overline{y}z + xy\overline{z}$$

   No credit will be given for solutions that use the truth table method.

$LHS = (z+x)(z+y)(\overline{y}+\overline{z})$	Commutative
$= (z+xy)(\overline{y}+\overline{z})$	Distributive
$= z\overline{y} + xy\overline{y} + z\overline{z} + xy\overline{z}$	Distributive
$= \overline{y}z + x \cdot 0 + 0 + xy\overline{z}$	Complement
$= \overline{y}z + 0 + 0 + xy\overline{z}$	Null Elements
$= \overline{y}z + xy\overline{z} = RHS$	Identity

3. (5 points) Counters.

For the binary ripple counter pictured below, fill in the following table which shows the values of $Q_0, Q_1, Q_2, Q_3, C_0, C_1, C_2, C_3$ during each clock pulse. Assume count enable is set to 1.



C_0	Q_0	C_1	Q_1	C_2	Q_2	C_3	Q_3
0	0	1	0	1	0	1	0
1	1	0	0	1	0	1	0
0	1	0	0	1	0	1	0
1	0	1	1	0	0	1	0
0	0	1	1	0	0	1	0
1	1	0	1	0	0	1	0
0	1	0	1	0	0	1	0
1	0	1	0	1	1	0	0
0	0	1	0	1	1	0	0
1	1	0	0	1	1	0	0

4. (5 points) State Diagram.

Draw the state diagram of a *minimal* Mealy machine having two input lines x, y, in which the signals $\{0, 1\}$ are applied, and a single output line z. For $i = 1, 2, 3, 4, \ldots$ let x_i, y_i denote the i-th input values. For $i \geq 1$, the system is to produce an output of 1 coincident with input symbols x_i, y_i if the binary number represented by $x_i, \ldots, x_1$ is greater than the binary number $y_i, \ldots, y_1$, where x_i and y_i are the most significant bits and x_1 and y_1 are the least significant bits. At all other times the system is to output 0.

An example of input/output sequences that satisfy the conditions of the system specification is:

i	1	2	3	4	5	6	7	8	9	10	11
x	1	0	1	1	1	0	1	0	1	1	0
y	1	1	1	0	0	1	1	0	0	1	0
z	0	0	0	1	1	0	0	0	1	1	1

In the example above, the system produces an output of 1 coincident with the 4-th input symbol. This occurs since the 1-st, 2-nd, 3-rd and 4-th input symbols are $1, 0, 1, 1$ on the x input line and $1, 1, 1, 0$ on the y input line, since $1101 > 0111$, the system outputs 1.

Your state diagram should have the minimum number of states possible.