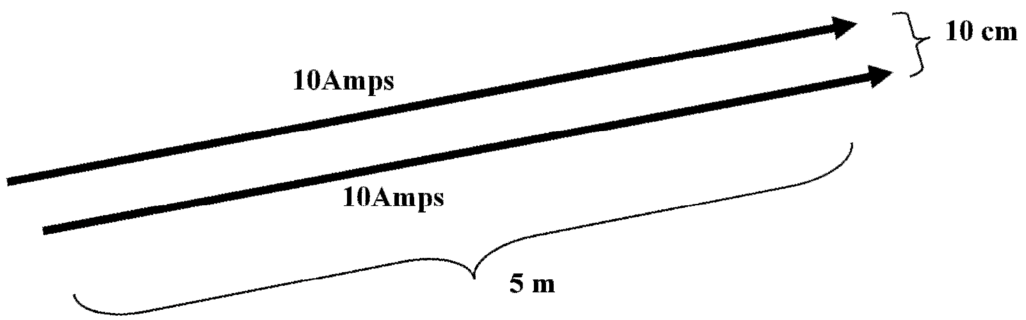
Solutions

1. (6) 10 Amperes of current are flowing in the same direction in each of two wires that are separated by 10cm. The wires are 5 m long.
What is the magnitude and direction of the force on each wire. Ignore the end effects

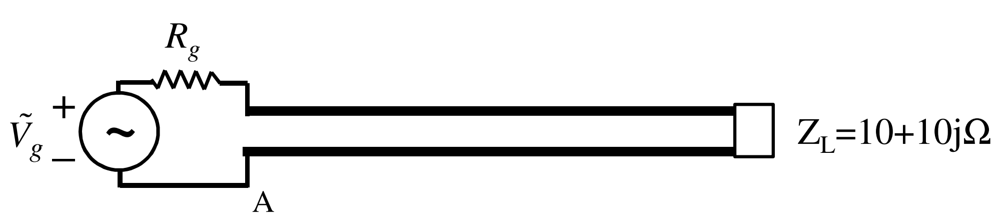


Magnetic field at the other wire is $H = \frac{I}{2\pi\rho} = \frac{10}{2\cdot\pi\cdot 0.1} = \frac{50}{\pi}$ Amp/meter

Force is $F = lI \times B = lI\mu_0 H = 5\cdot 10\cdot 4\pi\cdot 10^{-7}\cdot\frac{50}{\pi} = \ 10^{-3}$ Newtons

Attractive force

2. (7pts) An AC voltage generator with output impedance of $\boldsymbol{R_g}$**=10Ω**, peak amplitude of V_g **=100 Volts** operating at **300 MHz** is connected to a two-wire transmission line with characteristic impedance of **300Ω.**
The transmission line is **25 cm** long and is terminated with a load with impedance $\boldsymbol{Z_L}$ **=10+10j Ω.**
The transmission line is imbedded in a dielectric material of with $\boldsymbol{\varepsilon_r = 4}$ and $\boldsymbol{\mu_r = 1}$**.**



a. Calculate the phasors for **voltage** and **current** at input to the transmission line (point A).

The wavelength in vacuum is $\lambda_0 = \frac{c}{300MHz} = \frac{3\cdot 10^8 m/\sec}{3\cdot 10^8 /\sec} = 1 meter$

In the transmission line $\lambda = \frac{\lambda_0}{\sqrt{\varepsilon_r}} = \frac{\lambda_0}{2} = 50cm$

The transmission line length is $\frac{\lambda}{2}$. *It is the equivalent to* ***no*** *transmission line.*

$$\tilde{V}_A = V_g \frac{Z_L}{Z_L + R_g} = 100\frac{10+10j}{20+10j} = 100\frac{1+j}{2+j} = \frac{100}{5}(1+j)(1-2j) = 20(3-j) = 60+20j \ \textit{Volts}$$

$$\tilde{I}_A = V_g \frac{1}{Z_L + R_g} = 100\frac{1}{20+10j} = 10\frac{1}{2+1j} = \frac{10}{5}(2-j) = 2(2-j) = 4-2j \ \textit{Amps}$$

b. What is the average power going into the transmission line?

$$P_{Ave} = \frac{1}{2}\mathrm{Re}\left(\tilde{V}_A \tilde{I}^*\right) = \frac{1}{2}\mathrm{Re}\left\{(60+20j)(4+2j)\right\} = \frac{1}{2}\mathrm{Re}\left\{200+200j\right\} = 100\,Watt$$

3.(7pts) In a parallel plate waveguide with plate separation b, the z component of E field for TM_1 mode at radial frequency ω, given by phasor

$$\boldsymbol{E}_z(y,z) = A\sin\left(\frac{\pi y}{b}\right)e^{-j\beta z}$$

Waveguide is infinite in x direction; there is no variation of the fields in x direction. The plates are ideal conductors and the medium between them is vacuum.

a. What is the relationship between the frequency ω and the propagation constant β?

In general, for a waveguide, we can write the fields in the form

$$\boldsymbol{E}(x,y,z) = \boldsymbol{E}^0(x,y,z)e^{-j\beta z}$$

$$\text{and} \quad \boldsymbol{H}(x,y,z) = \boldsymbol{H}^0(x,y,z)e^{-j\beta z}$$

The fields satisfy the wave equation: $\vec{\nabla}^2\boldsymbol{E} + k^2\boldsymbol{E} = 0$

$$\nabla_{xy}^2 E^0(x,y) + \left(-\beta^2 + k^2\right)E^0(x,y) = 0$$

For this mode $h^2 = -\beta^2 + k^2 = \left(\frac{\pi}{b}\right)^2 \qquad -\beta^2 + \omega^2\mu\varepsilon = \left(\frac{\pi}{b}\right)^2 \qquad \beta = \sqrt{\omega^2\mu\varepsilon - \left(\frac{\pi}{b}\right)^2}$

b. Calculate expressions for phasors for all the non-zero components of the E and H fields for this mode.

These fields are related by the Maxwell's equations $\nabla\times\boldsymbol{E} = -j\omega\mu\boldsymbol{H}$ and $\nabla\times\boldsymbol{H} = j\omega\varepsilon\boldsymbol{E}$

$$\nabla\times\boldsymbol{E} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E_x^0(x,y)e^{-\gamma z} & E_y^0(x,y)e^{-\gamma z} & E_z^0(x,y)e^{-\gamma z} \end{vmatrix}$$

$$\nabla\times H = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ H_x^0(x,y)e^{-\gamma z} & H_y^0(x,y)e^{-\gamma z} & H_z^0(x,y)e^{-\gamma z} \end{vmatrix}$$

We have six equations relating different field components.

$$\frac{\partial}{\partial y}E_z^0 + \gamma E_y^0 = -j\omega\mu H_x^0 \qquad -\gamma E_x^0 - \frac{\partial}{\partial x}E_z^0 = -j\omega\mu H_y^0 \qquad \frac{\partial}{\partial y}E_x^0 - \frac{\partial}{\partial x}E_y^0 = -j\omega\mu H_z^0$$

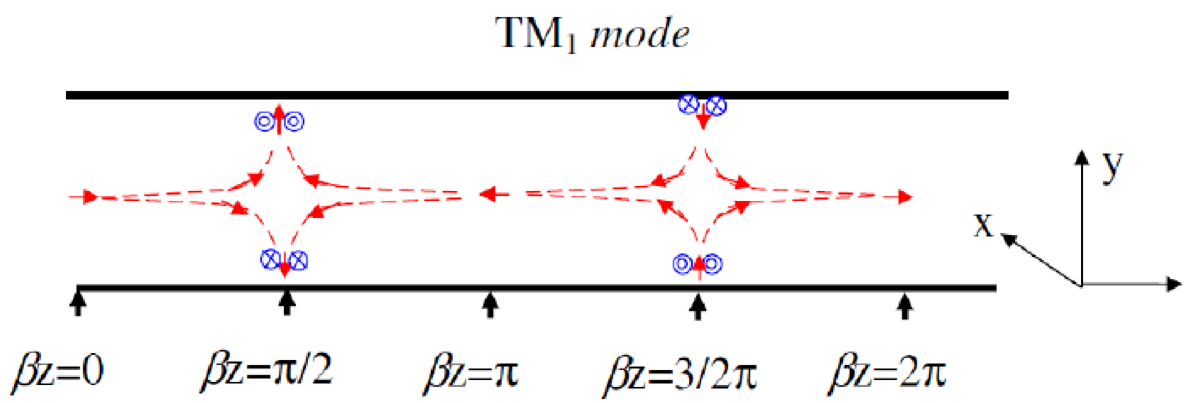
$$\frac{\partial}{\partial y}H_z^0 + \gamma H_y^0 = j\omega\varepsilon E_x^0 \qquad -\gamma H_x^0 - \frac{\partial}{\partial x}H_z^0 = j\omega\varepsilon E_y^0 \qquad \frac{\partial}{\partial y}H_x^0 - \frac{\partial}{\partial x}H_y^0 = j\omega\varepsilon E_z^0$$

TM mode has $H_z^0 = 0$, and derivative of each field component with respect to x is zero. From these equations we get $E_x^0 = 0$, $H_y^0 = 0$ and

$$H_x^0 = \frac{j\omega\varepsilon}{\gamma^2 + \omega^2\mu\varepsilon}\frac{\pi}{b}A\cos\left(\frac{\pi y}{b}\right) = \frac{j\omega\varepsilon}{h}A\cos\left(\frac{\pi y}{b}\right)$$
$$E_y^0 = \frac{-j\beta}{\gamma^2 + \omega^2\mu\varepsilon}\frac{\pi}{b}A\cos\left(\frac{\pi y}{b}\right) = \frac{-j\beta}{h}A\cos\left(\frac{\pi y}{b}\right)$$

c. Write down the expressions for all the real non-zero E field components for TM_1 at t=0, and sketch the shape of the E field lines for TM_1 mode

$$E_z(y,z) = A\sin\left(\frac{\pi y}{b}\right)\cos(\beta z)$$
$$E_y = \text{Re}\left\{-\frac{j\beta}{h}A\cos\left(\frac{\pi y}{b}\right)e^{-j\beta z}\right\} = \text{Re}\left\{-\frac{j\beta}{h}A\cos\left(\frac{\pi y}{b}\right)(\cos\beta z - j\sin\beta z)\right\} = -\frac{\beta}{h}A\cos\left(\frac{\pi y}{b}\right)\sin\beta z$$



Electric fields are shown with dashed red lines