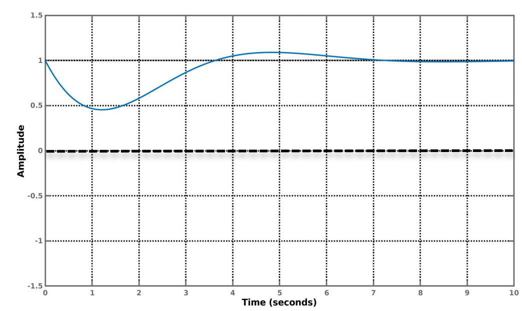
Part 1 (6 pts)

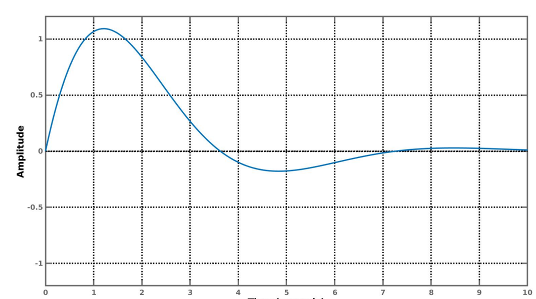
In each part of this problem, the step response shown corresponds to one of the following system functions:

$$H_1(s) = \frac{2s}{s^2 + s + 1} \qquad H_2(s) = \frac{s^2 + 1}{s^2 + s + 1} \qquad H_3(s) = \frac{2s}{s^4 + s^2 + 1}$$

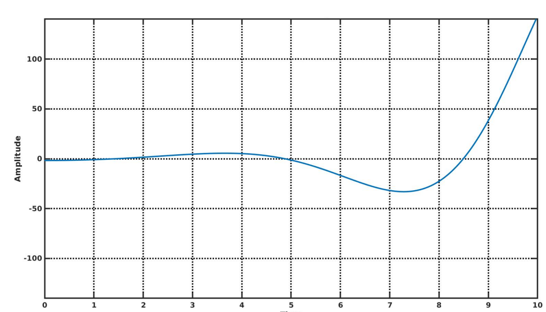
(a) In the *step response* shown to the right, the graph seems to level out at about y=1. Is y=1 the exact final value as t goes to infinity? If so, **explain why**. If not, give the **actual limiting value** of y.



(b) In the *step response* shown to the right, the curve appears to cross the t-axis at about t=3.5. Is this the exact value at which the crossing occurs? If so, **explain why**. If not, give the **actual value** at which the crossing occurs.



(c) In the *step response* shown to the right, the response appears not yet to have reached a steady state at t=10. If we extend the time scale, should we expect the response ultimately to reach a steady state? Or should we expect some other type of behavior? **Explain your answer**.



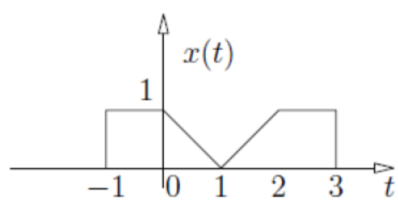
Part 2 (5 pts)

Find y(t)=x(t)*h(t) if $x(t) = \sum_{n=-\infty}^{\infty} \delta(t-n)$ and the FT of $h(t)$,

$$H(\omega) = \begin{cases} j\sin(\frac{\omega}{4}), & |\omega| < 4\pi \\ 0, & otherwise \end{cases}$$

Part 3 (4 pts).

Let X(ω) be the Fourier transform of the signal x(t) shown below.



Sketch the inverse Fourier transform of Re$\{X(\omega)\}$ without explicitly finding $X(\omega)$.

Part 4. (5 pts) Find $x[n]$ based on the following four facts:

(a) $x[n] = 0$ for $n > 0$
(b) $x[0] > 0$
(c) $Im\{X(\Omega)\} = sin\Omega - sin2\Omega$
(d) $\frac{1}{2\pi}\int_{-\pi}^{\pi} |X(\Omega)|^2 d\Omega = 3$

Answers:

Part 1.

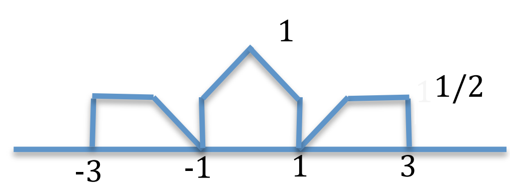
(a) Only $H_2(s)$ has a value at $s = 0$ that is not zero, and that value is 1. So, the limiting value is precisely 1.

(b) The step response dies away so that it is due to a stable system. Thus, it must be due to $H_1(s)$. The step response is the inverse of $\frac{1}{s}H_1(s) = \frac{2}{s^2+s+1}$ which has poles at $s = -\frac{1}{2} \pm \frac{\sqrt{3}}{2}$ giving $y(t) = \frac{4}{\sqrt{3}} e^{-t/2} sin\frac{\sqrt{3}}{2}t, t > 0$. $y(t)$ has it's first zero at $\frac{\sqrt{3}}{2}t = \pi$ or $t = \frac{2\pi}{\sqrt{3}} \neq 3.5$

(c) This is the step response of $H_3(s)$ which is unstable with poles at the root of $s^4 + s^2 + 1$, or at $s^2 = -\frac{1}{2} \pm j\frac{\sqrt{3}}{2}$ which gives roots at $s = \pm\frac{1}{2} \pm j\frac{\sqrt{3}}{2}$. Right half plane poles correspond to growing exponentials so no steady state will be reached.

Part 2. $X(\omega) = 2\pi \sum_{k=-\infty}^{\infty} \delta(\omega - 2\pi k)$ and
$H(\omega)X(\omega) = 2\pi j sin(\frac{\pi}{2})\delta(\omega - 2\pi) + 2\pi j sin(-\frac{\pi}{2})\delta(\omega + 2\pi)$ so that
$y(t) = -2sin2\pi t$

Part (3). $Re\{X(\omega)\} \leftrightarrow x_{even}(t) = \frac{x(t)+x(-t)}{2}$



Part 4. $jIm\{X(\Omega\} \leftrightarrow x_{odd}[n] = \frac{1}{2}(-\delta[n+2] + \delta[n+1] - \delta[n-1] + \delta[n-2]) = \frac{x[n]-x[-n]}{2}$, given $x[n] = 0, n > 0$ then $x[n] = 2x_{odd}[n]$ for $n < 0$. Finally, using Parseval's theorem, we know that $\sum_{n=-\infty}^{\infty} x^2[n] = 3$ so that $x[0] = 1$. Thus, $x[n] = -\delta[n+2] + \delta[n+1] + \delta[n]$.