Qual exam ECE/ Basic Physics: Spring 2018

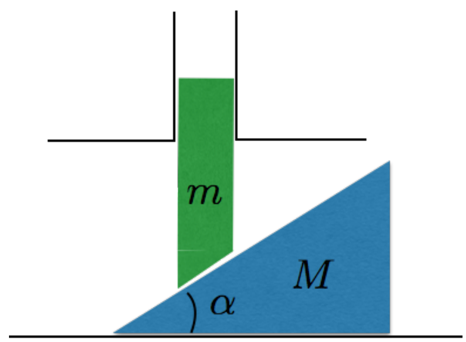
1. Sliding blocks (6 pts)

A wedge of mass m is located on another wedge of mass M and angle α. The top edge is confined in the vertical direction. We ignore friction in this problem.

(a) Show all forces on both blocks, i.e., the force diagram. a_1 is the acceleration of M with respect to the surface and a_1 is the vertical acceleration of m. Indicate a_1 and a_2 on the diagram. (1 pts)

(b) Using the force diagram from part (a) find the relation between acceleration and force, i.e., equations of motions. Also, find a geometrical relation between a_1 and a_2 (3pts)

(c) Find a_1 and a_2. (1pts)



2. Two blocks and a spring (7 pts)

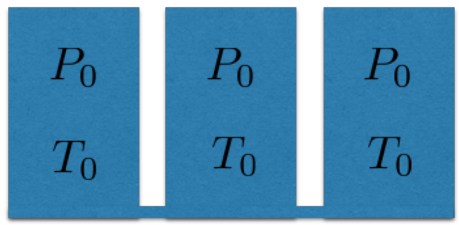
Two blocked are connected by a massless spring of stiffness k and length (in the non-deformed state) l_0 rest on a horizontal plane. A constant horizontal force F pulls one of the blocks.

(a) Show all the forces on the blocks and write the equation of motions (3 pts)

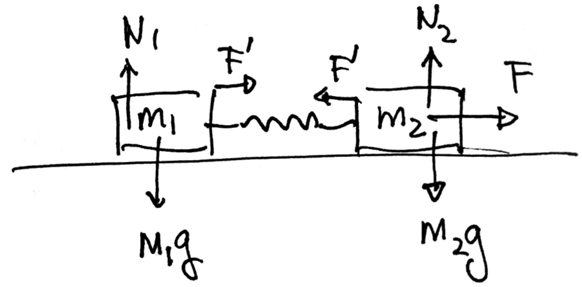
(b) Find the maximum distance between the blocks during this motions (4 pts)

3. Gas containers (7 pts)

Three identical gas containers are connected with a thin tube. The initial temperature of all of them is T_0 and the pressure is P_0. We raise the temperature of the left chamber by ΔT and lower the temperature of the right one by ΔT.
(a) Assuming that the heat conduction is negligible, what is the new pressure in each container? (4 pts)
(b) How much does the internal energy of the whole system changes? (3 pts)



$$
\begin{cases}
m_1 \ddot{x}_1 = F' \\
m_2 \ddot{x}_2 = F - F' \\
F' = (x_2 - x_1 - l_0) k
\end{cases}
$$

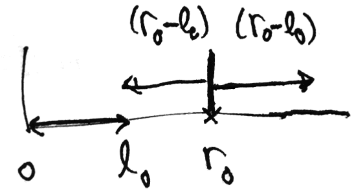


$$\rightarrow \ddot{x}_2 - \ddot{x}_1 = \frac{F}{m_2} - k\left(\frac{1}{m_1} + \frac{1}{m_2}\right)(x_2 - x_1 - l_0)$$

defining $r = x_2 - x_1$ and $r_0 = l_0 + \frac{F}{k} \frac{m_1}{m_1 + m_2}$

$$\ddot{r} = -\frac{k}{m_1}(r - r_0)$$

The motion in relative coordinates is that of a displaced harmonic oscillator, centered at r_0. The initial value of r is l_0

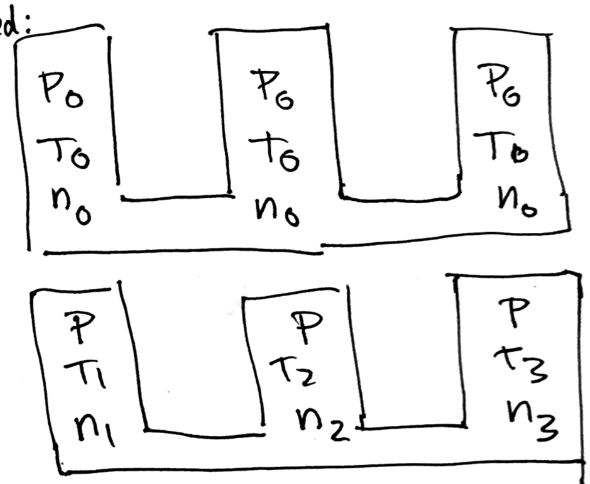


So the amplitude of oscillation is $r_0 - l_0 = \frac{F}{k} \frac{m_1}{m_1 + m_2}$

$$\Rightarrow \text{max. separation} = l_0 + \frac{2F}{k} \frac{m_1}{m_1 + m_2}$$

$$\text{min. separation} = l_0$$

Total number of molecules is conserved:



$$n_1+n_2+n_3 = 3n_0$$

$$P_0V = n_0RT_0$$

$$PV = n_1RT_1 = n_2RT_2 = n_3RT_3$$

$$\rightarrow \frac{PV}{R}\left(\frac{1}{T_1}+\frac{1}{T_2}+\frac{1}{T_3}\right) = 3n_0$$

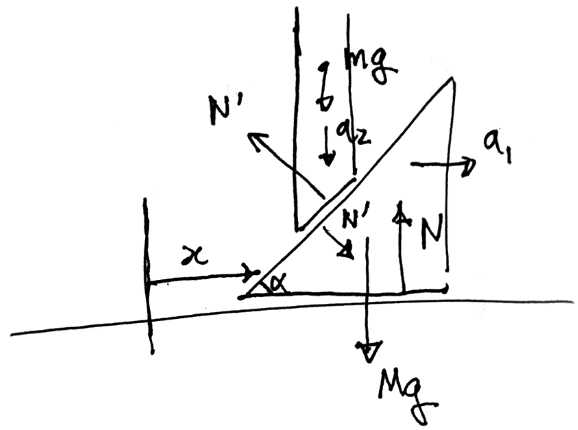
$$P = \frac{3n_0R}{V}\left(1/T_1+1/T_2+1/T_3\right)^{-1} = \frac{3P_0}{T_0} \; \frac{T_1T_2T_3}{T_1T_2+T_2T_3+T_3T_1}$$

$$P = 3P_0\left(\frac{T_0^2-\Delta T^2}{3T_0^2-\Delta T^2}\right)$$

Total internal energy: $U = U_1+U_2+U_3$

$$= C_v\,(n_1T_1+n_2T_2+n_3T_3)$$

$$U = 3C_v\frac{PV}{R} \Rightarrow \Delta U = \frac{3C_v}{R}V(P-P_0) = \frac{3C_vV}{R}\;\frac{-2\Delta T^2}{3T_0^2-\Delta T^2} < 0$$



$$\frac{a_2}{a_1} = \tan\alpha$$

$$\begin{cases} M a_1 = N' \sin\alpha \\ m a_2 = mg - N' \cos\alpha \end{cases}$$

$$m a_2 = mg - \cot\alpha \, M a_1$$

$$m a_1 \tan\alpha = mg - \cot\alpha \, M a_1$$

$$\rightarrow \quad a_1 = \frac{g}{\tan\alpha + \cot\alpha \, M/m}$$

$$a_2 = \frac{g}{1 + \cot^2\alpha \, \frac{M}{m}}$$