ECE Written Qualifying Examination, Circuits Spring 2018

1. 7 points

The following NMOS circuit is biased for drain current I_D=1mA using the battery of voltage V_{DD}=9V. Assume the transistor is described by

$$I_D=k(V_{GS}-Vth)^2(1+\lambda V_{DS})$$

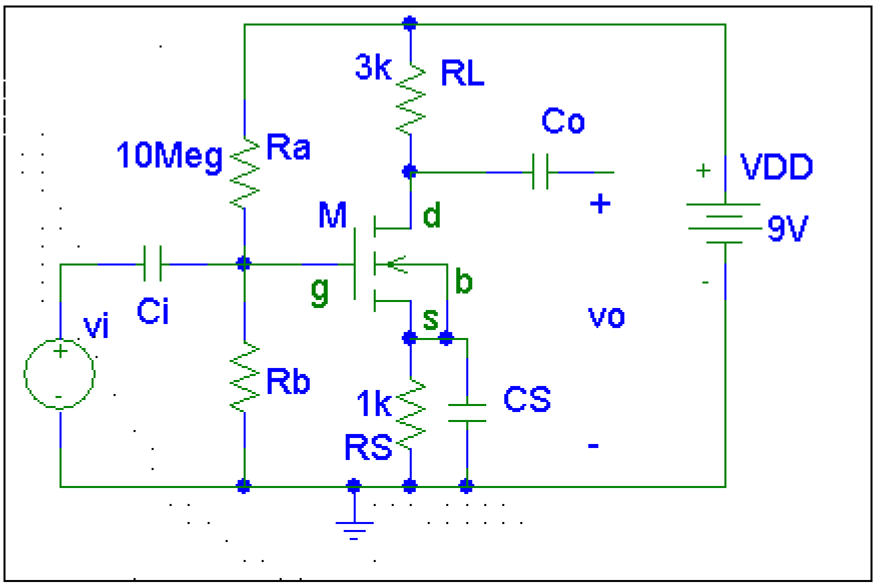
with

$k=10^{-3}$, Vth=1 and $\lambda=0.2$ (use output conductance $g_o=\lambda I_D$)

a) Determine V_{DS} at bias.

b) Determine V_{GS} and Rb at bias.

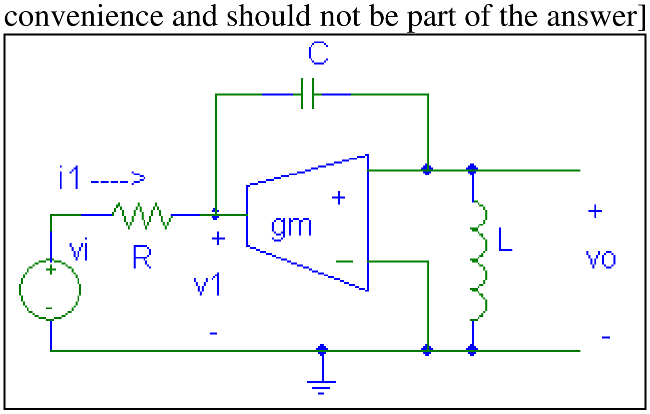
c) With frequencies such that all capacitors shown act as shorts, give the small signal voltage gain, vo/vi.



2. 7 points

An Operational Transconductance Amplifier (OTA) is described by the property that no current flows into the input (+ & - terminals) and has output current (directed into the OTA) as $io=gm(v_+ - v_-)$ where gm is the transconductance.

For the following OTA circuit find the open circuit voltage gain, vo(s)/vi(s), and discuss the possibility of this forming an oscillator. [note that the node voltage V1 is labelled for convenience and should not be part of the answer].



3. 6 points

A tunnel diode is described by its current i_d versus voltage v_d by the equation

$$i_d=0.1v_d(v_d^2 - 10.5v_d + 30)$$

for which the derivative is $di_d/dv_d=0.3(v_d-2)(v_d-5)$. It has a load current y given by

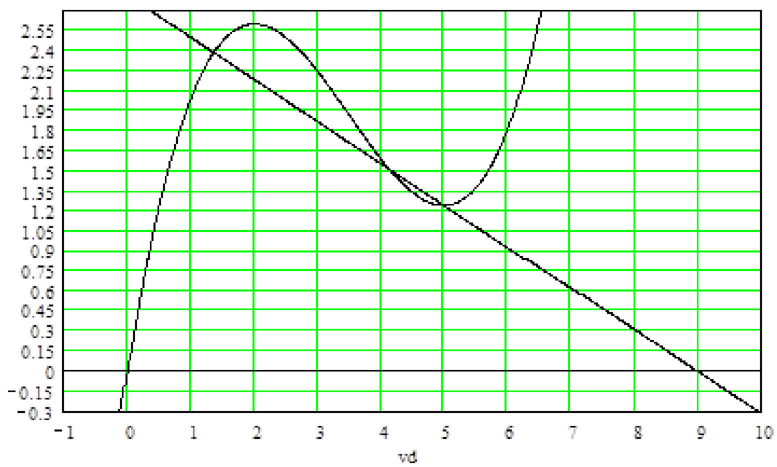
$$y=m.v_d + Yo$$

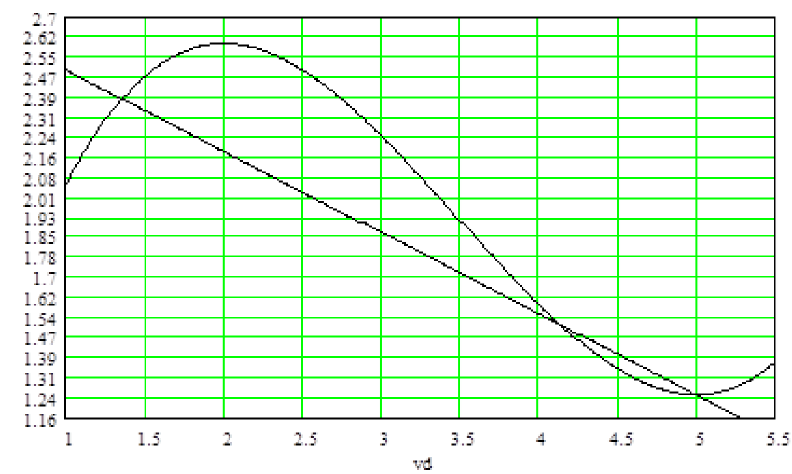
These curves are plotted below [followed by the bottom figure giving detail near the intersect points.

a) Determine m and Yo for the load line (given that it passes through the local minimum of i_d at v_d=5).

b) Give the current and voltage values at the middle intersect point.

c) Determine the small signal conductance at the middle intersect point.





solutions, circuits 2018 spring

#1. a) by KVL: $V_{DS} = V_{DD} - (R_L + R_S) I_D = 9 - (3+1)\times 10^3 \times 1\times 10^{-3} = 9 - 4 = \underline{5V = V_{DS}}$

b) $I_D = 1\times 10^{-3} = 1\times 10^{-3}(V_{GS}-1)^2(1+0.2\times 5) \Rightarrow 1 = (V_{GS}-1)^2 \times 2 \Rightarrow V_{GS} = 1 \pm 1/\sqrt{2}$

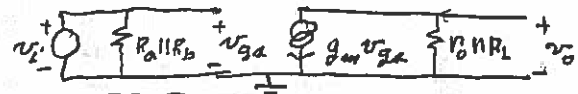
$\Rightarrow \underline{V_{GS} = 1 + 1/\sqrt{2}}$ (use + sign as $V_{GS} > 1$ for $I_D > 0$)

For R_b: $V_G = \frac{R_b}{R_a + R_b} \cdot V_{DD} = V_{GS} + R_S I_D = 1 + \frac{1}{\sqrt{2}} + 10^3 \times 10^{-3} = 2 + 1/\sqrt{2} = 2.707$

$\Rightarrow (2 + 1/\sqrt{2})(R_a + R_b) = R_b \cdot 9 \Rightarrow (2 + \frac{1}{\sqrt{2}}) R_a = (7 - \frac{1}{\sqrt{2}}) R_b$

$\Rightarrow R_b = (\frac{2\sqrt{2}+1}{7\sqrt{2}-1}) R_a = \frac{3.828}{8.90} R_a = 0.434 R_a \Rightarrow \underline{R_b = 4.34\,MEG}$

c) small signal equivalent



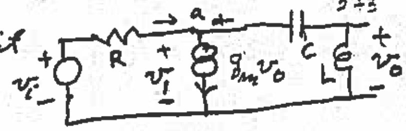
$g_m = \frac{\partial i_D}{\partial v_{GS}} = \frac{2 I_D}{(V_{GS} - V_{th})} = \frac{2\times 10^{-3}}{(1 + 1/\sqrt{2} - 1)} = \boxed{2\times\sqrt{2}\times 10^{-3} = g_m} = 2.83\times 10^{-3}\,℧$

assumption of prob: $g_{o_1} = \lambda I_D = 0.2\times 10^{-3} = 2\times 10^{-4} \Rightarrow r_{o_1} = 10^4/2 = 5\times 10^3$

more accurate $g_{o_2} = \frac{\partial I_D}{\partial V_{DS}} = k(V_{GS} - V_{th})^2 \lambda = 10^{-3}(1 + 1/\sqrt{2} - 1)^2 \times 0.2 = \frac{1}{2}\times 2\times 10^{-4} = 1\times 10^{-4} \Rightarrow r_{o_2} = 10\times 10^3$

∴ $v_o/v_i = -g_m (r_o \| R_L) \approx -2\sqrt{2}\times 10^{-3} \times (\frac{5\times 3}{5+3})\times 10^3 = -2\sqrt{2}\times\frac{15}{8} = -5.3$ for r_{o_1} or $-2\sqrt{2}\times\frac{3\times 10}{3+10} = -6.53$ for r_{o_2}

#2 Equivalent circuit



KCL@a: $G(v_i - v_1) - g_m v_o + sC(v_o - v_1) = 0$, $G = 1/R \Rightarrow$ v_1 in terms of v_i & $v_o \Rightarrow Gv_i + (sC - g_m)v_o = (G+sC)\cdot$

$\Rightarrow v_1 = [G v_i + (sC - g_m) v_o]/[G + sC]$ also by voltage divider $v_o = \frac{sC}{(sL + 1/sC)} v_1$

$\Rightarrow v_1 = (\frac{s^2 LC + 1}{s^2 LC}) v_o$; equate v_1's: $(s^2 LC + 1)(G + sC) v_o = s^2 LC [G v_i + (sC - g_m) v_o]$

$\Rightarrow \frac{v_o}{v_i} = s^2 LCG / [LC(G + g_m)s^2 + Cs + G] = \underline{\underline{\frac{s^2 LC}{LC(1 + g_m R)s^2 + RCs + 1}}} = v_o/v_i$

$\Rightarrow$ oscillator if $RC = 0 \Rightarrow R = 0$ (as $C = 0 \Rightarrow v_o = 0$) to give a pole @ $LCs^2 + 1 = 0 \Rightarrow \omega_0 = 1/\sqrt{LC}$

#3 a) For the load line need 2 points: $i = 0$ @ $v = 9$, @ $v = 5$, $i = 0.1\times 5(25 - 52.5 + 30) = 1.25$

$y = 0 = m\times 9 + Y_0$, $y = 1.25 = m\times 5 + Y_0$; subtract $1.25 = m(5 - 9) \Rightarrow m = -1.25/4 = -5/16 = \underline{-0.3125 = m}$

$\Rightarrow Y_0 = +\frac{1.25}{4}\times 9 = \underline{2.8125 = Y_0} = 45/16$

b) @ the middle intersect point $0.1 v_d(v_d^2 - 10.5 v_d + 30) = -0.3125 v_d + 2.8125$

which is a cubic with a known zero @ $v_d = 5 \Rightarrow v_d^3 - 10.5 v_d^2 + 33.125 v_d - 28.125 = (v_d - 5)(v_d^2 + a v_d + b)$

Equating coefficients $-5b = -28.125$, $-10.5 = -5 + a \Rightarrow a = -5.5$, $b = 5.625$

$v_{d_{2,3}} = \frac{5.5}{2} \pm \frac{1}{2}\sqrt{(5.5)^2 - 4\times 5.625} = (5.5 \pm \sqrt{30.25 - 22.5})/2 = (5.5 \pm 2.784)/2 = 4.142$ or 1.358

∴ middle $\underline{v_d = 4.142}$ and $i = -0.3125\times 4.142 + 2.8125 = \underline{1.518 = i_d}$ (from curves we see $v_d \approx 4.2$, $i_d = 1.5$)

c) $g = \frac{\partial i_d}{\partial v_d} = 0.3(v_d - 2)(v_d - 5)\Big|_{\text{intersect}} \approx 0.3(4.14 - 2)(4.14 - 5) = \underline{-0.62\,℧}$

(using guess from curve $= 0.3(4.2 - 2)(4.2 - 5) = 0.3\times 2.2\times(-0.8) = -0.52\,℧$)