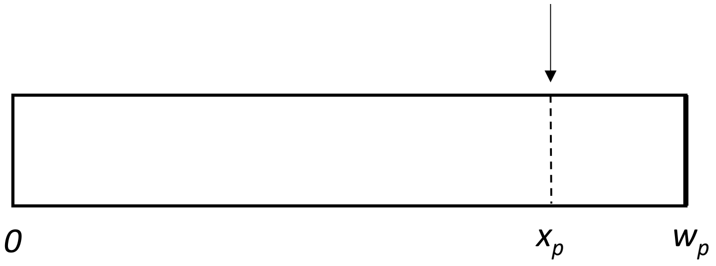
**Problem 1 – Carrier Diffusion**



Consider a uniformly-doped, p-type bar of silicon of length $w_p$ with an ohmic contact on the right-hand side at $x = w_p$. Light is projected along the dotted line at $x = x_p$ and this produces electron-hole pairs at a rate of $G_L$.

Assume that the minority carrier diffusion length is very long compared to $w_p$. Assume that you know the minority carrier diffusion constant D and recombination length constant L.

a) (4 pts) What are the equations for the **excess** minority carrier concentration in the two regions $0 < x < x_p$ and $x_p < x < w_p$? Be sure to give your answer in terms of $G_L$.

b) (2 pts) What is the equation for the excess **majority** carrier concentration in the two regions $0 < x < x_p$ and $x_p < x < w_p$?

**Problem 2 – PN-junction diode**

The equation for the pn-junction diode is given as: $I_D = I_S\left(e^{\frac{V_D}{V_T}} - 1\right)$ where $I_S$ is a scaling current that is a function of the parameters of the diode such as doping concentrations and the geometry of the diode.

a) (3 pts) Provide a **short**, intuitive explanation of the '-1' term. (Hint: Explain this in terms of the minority carrier concentrations on both sides of the depletion region.)
b) (4 pts) In forward-bias operation, the minority carrier concentrations are above the equilibrium concentration outside the depletion region (aka "quasineutral" regions), so the diode must be constructed so that the p and n regions need to extend well beyond the depletion region. How do the doping concentrations affect the size of the "active" portion (depletion region & quasineutral p and n regions) of the diode?

**Problem 3 – n-channel MOSFET**

The standard formula for the above-threshold, saturation-mode, drain current in the nFET is: $I_D = \frac{1}{2}k_n' \frac{W}{L}(V_{GS} - V_t)^2$ where $V_{GS}$ is the gate to source voltage, $V_t$ is the threshold voltage, $k_n'$ is the process transconductance, and $W/L$ is the width to length ratio. In the triode mode of operation, the drain current equation is given as:

$$I_D = k_n' \frac{W}{L}\left[(V_{GS} - V_t)V_{DS} - \frac{1}{2}V_{DS}^2\right]$$

a) (3 pts) Provide a short, intuitive explanation for why in saturation, the current is quadratic in $(V_{GS} - V_t)$. (Hint: think in terms of the transconductance of the channel.)
b) (4 pts) The nFET is occasionally used in the triode mode of operation as a resistance. Solve for the small signal (aka, "incremental") drain conductance $\frac{i_d}{v_{ds}}$ at a given $V_{DS}$ and $V_{GS}$, assuming that we are operating in triode. For what $V_{DS}$ does this small signal conductance go to zero?

1a) The first region does not have an outlet for minority carriers and because $x_p$ is small compared to the diffusion length, there is no significant loss of minority carriers due to recombination. We adopt the linear solution with the boundary condition $\frac{dn'(0)}{dx} = 0$. This means that *n'(x)* is constant and > 0 in the first region. Thus, $n'(x) = n'(x_p)$ for $0 < x < x_p$. We will solve for n'($x_n$) after the next section.

In the second region, we have an ohmic contact on the right and again, $w_p - x_p \ll L_e$, so we use the simple linear solution $n'(x) = n'(x_p) \cdot \left( \frac{w_p - x}{w_p - x_p} \right)$ for $x_p < x < w_p$

To find $n'(x_p)$, we need to satisfy the conservation of flux. In this problem, since the carriers are only being lost to the right, $G_L$ = the flux going towards the right,

$$G_L = -D_n \frac{dn'(x_p^{+})}{dx}$$

$$G_L = D_n \frac{n'(x_p)}{w_p - x_p} \quad \text{so,} \quad n'(x_p) = \frac{G_L\left(w_p - x_p\right)}{D_n}$$

$$\boxed{n'(x) = \frac{G_L\left(w_p - x_p\right)}{D_n}} \text{ for } 0 < x < x_p \quad \text{and}$$

$$\boxed{n'(x) = \frac{G_L\left(w_p - x_p\right)}{D_n} \frac{\left(w_p - x\right)}{\left(w_p - x_p\right)} = \frac{G_L}{D_n}\left(w_p - x\right)} \text{ for } x_p < x < w_p$$

1b) Since we can argue for quasineutrality to hold in this region, $p'(x) \approx n'(x)$

$$\boxed{p'(x) = \frac{G_L\left(w_p - x_p\right)}{D_n}} \text{ for } 0 < x < x_p$$

$$\boxed{p'(x) = \frac{G_L\left(w_p - x_p\right)}{D_n} \frac{\left(w_p - x\right)}{\left(w_p - x_p\right)} = \frac{G_L}{D_n}\left(w_p - x\right)} \text{ for } x_p < x < w_p$$

2a) The diode current is derived from looking at the gradient of the minority carrier concentration (diffusion and recombination) on either side of the depletion region edge. For example, looking at the n-side, the concentration of holes (minority carrier) at the edge is proportional to $\frac{n_i^2}{N_D} e^{\frac{V_D}{V_T}}$ and it decreases exponentially to the equilibrium level of $\frac{n_i^2}{N_D}$. This means that the height of the exponential is: $\frac{n_i^2}{N_D} e^{\frac{V_D}{V_T}} - \frac{n_i^2}{N_D} = \frac{n_i^2}{N_D}\left( e^{\frac{V_D}{V_T}} - 1 \right)$ Likewise for the p-side.

2b) The doping concentrations in the n and p regions affect the sizes of both the non-equilibrium "quasineutral" regions and the depletion region. Increasing the doping concentrations reduces the depletion region width because it takes less width to contain the charge needed to create the built-in voltage. Increasing the doping concentrations also reduces the distance it takes for the minority carrier concentrations to diminish to their equilibrium value due to the increased rate of recombination.

3a) The nFET drain current is the product of the conductance of the channel times the voltage across that conductance. The conductance of the channel is controlled by $(V_{GS} - V_t)$ and the voltage at the drain end of the channel (pinch-off point) is also $(V_{GS} - V_t)$, so the drain current becomes quadratic in $(V_{GS} - V_t)$.

3b) To find $\frac{i_d}{v_{ds}}$ at a given $V_{DS}$, we need to take the derivative of $I_D = k_n' \frac{W}{L}\left[(V_{GS} - V_t)V_{DS} - \frac{1}{2}V_{DS}^{\ 2}\right]$ with respect to $V_{DS}$, for $V_{DS}$ near zero. $\frac{dI_D}{dV_{DS}} = \frac{di_d}{dv_{ds}} = k_n' \frac{W}{L}\left[(V_{GS} - V_t) - V_{DS}\right]$ for small $v_{ds}$

We can find where this small signal conductance goes to zero by solving the following equation for $V_{DS}$:

$\frac{di_d}{dv_{ds}} = k_n' \frac{W}{L}\left[(V_{GS} - V_t) - V_{DS}\right] = 0$ ➔ $V_{DS} = V_{GS} - V_t$ which is the transition between the triode and saturation modes.