ECE Written Qualifying Examination, Winter 2018
Digital Logic

1. (4 points) Boolean Simplification.

   Consider the function $f(w, x, y, z) = \sum m(0, 2, 4, 5, 6, 7, 8, 10, 15)$. Using a Karnaugh map, determine all the prime implicants of f. Find a minimal sum for f. What is the cost of this minimal sum (assume the cost criteria is the total number of gate inputs)?

2. (4 points) Boolean Algebra.

   Using Boolean Algebra postulates and theorems prove that

   $$x + (yx \oplus y) = x + y,$$

   where $\oplus$ denotes XOR. No credit will be given for solutions that use the truth table method.

3. (5 points) MSI Components.

Consider the Boolean expression $f(w, x, y, z) = \sum m(0, 2, 3, 7, 8, 10, 12, 13, 15)$. Realize this Boolean expression using two levels of 4-to-1 multiplexers and no logic gates.

4. (7 points) State Diagram.

Draw the state diagram of a *minimal* Mealy machine having a single input line x, in which the symbols 0 and 1 are applied, and a single output line z. For $i = 1, 2, 3, 4, \ldots$ let x_i denote the i-th input symbol. For $i \geq 4$, the system is to produce an output of 1 coincident with input symbol x_i if the previous 4 bits (from left to right) $(x_{i-3}, x_{i-2}, x_{i-1}, x_i)$ correspond to $0100, 1010$, or 1100. At all other times the system is to output 0. An example of input/output sequences that satisfy the conditions of the system specification is:

i	1	2	3	4	5	6	7	8	9	10	11
x	1	0	1	0	0	1	1	0	0	1	0
z	0	0	0	0	1	0	0	0	1	0	0

In the example above, the system produces an output of 1 coincident with the 4-th input symbol. This occurs since the 1-st, 2-nd, 3-rd and 4-th input symbols are $1, 0, 1, 0$, which matches the pattern.

Your state diagram should have the minimum number of states possible.

Solutions

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1. (4 points) Boolean Simplification.

   Consider the function $f(w,x,y,z) = \sum m(0,2,4,5,6,7,8,10,15)$. Using a Karnaugh map, determine all the prime implicants of f. Find a minimal sum for f. What is the cost of this minimal sum (assume the cost criteria is the total number of gate inputs)?

wx \ yz	00	01	11	10
00	1 (0)	0 (1)	0 (3)	1 (2)
01	1 (4)	1 (5)	1 (7)	1 (6)
11	0 (12)	0 (13)	1 (15)	0 (14)
10	1 (8)	0 (9)	0 (11)	1 (10)

prime implicants:

$\bar{y}\bar{z}$, $\bar{w}x$, xyz, $\bar{w}\bar{z}$

minimal sum:

$\bar{y}\bar{z} + \bar{w}x + xyz$

cost: 10

2. (4 points) Boolean Algebra.

   Using Boolean Algebra postulates and theorems prove that

$$x + (yx \oplus y) = x + y,$$

where $\oplus$ denotes XOR. No credit will be given for solutions that use the truth table method.

$yx \oplus y = (\bar{y} + \bar{x})y + yx\bar{y}$ definition of XOR

$= \bar{y}y + \bar{x}y + yx\bar{y}$ distributive

$= \bar{x}y$ complement, identity

$x + \bar{x}y = (x+\bar{x})(x+y)$ distributive

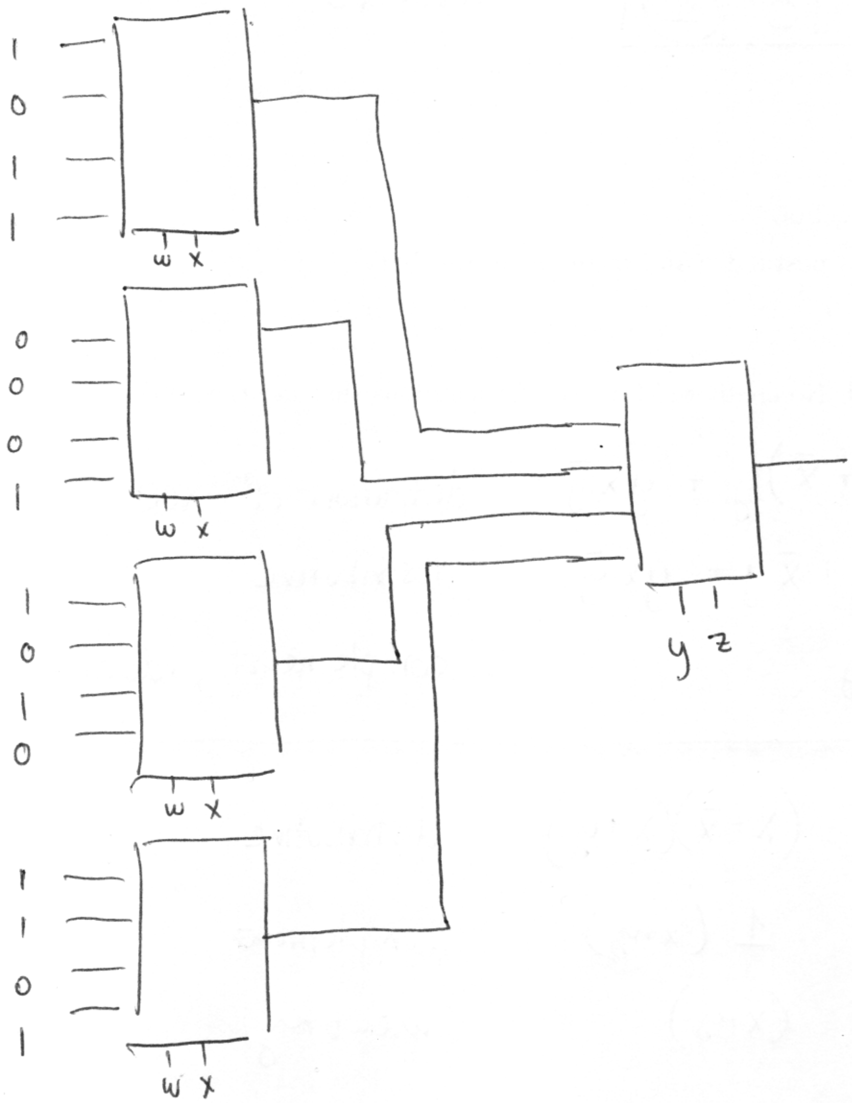
$= 1\,(x+y)$ complement

$= (x+y)$ identity

3. (5 points) MSI Components.

Consider the Boolean expression $f(w,x,y,z) = \sum m(0,2,3,7,8,10,12,13,15)$. Realize this Boolean expression using two levels of 4-to-1 multiplexers and no logic gates.

wx \ yz	00	01	11	10
00	1	0	1	1
01	0	0	1	0
11	1	1	1	0
10	1	0	0	1



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Your state diagram should have the minimum number of states possible.