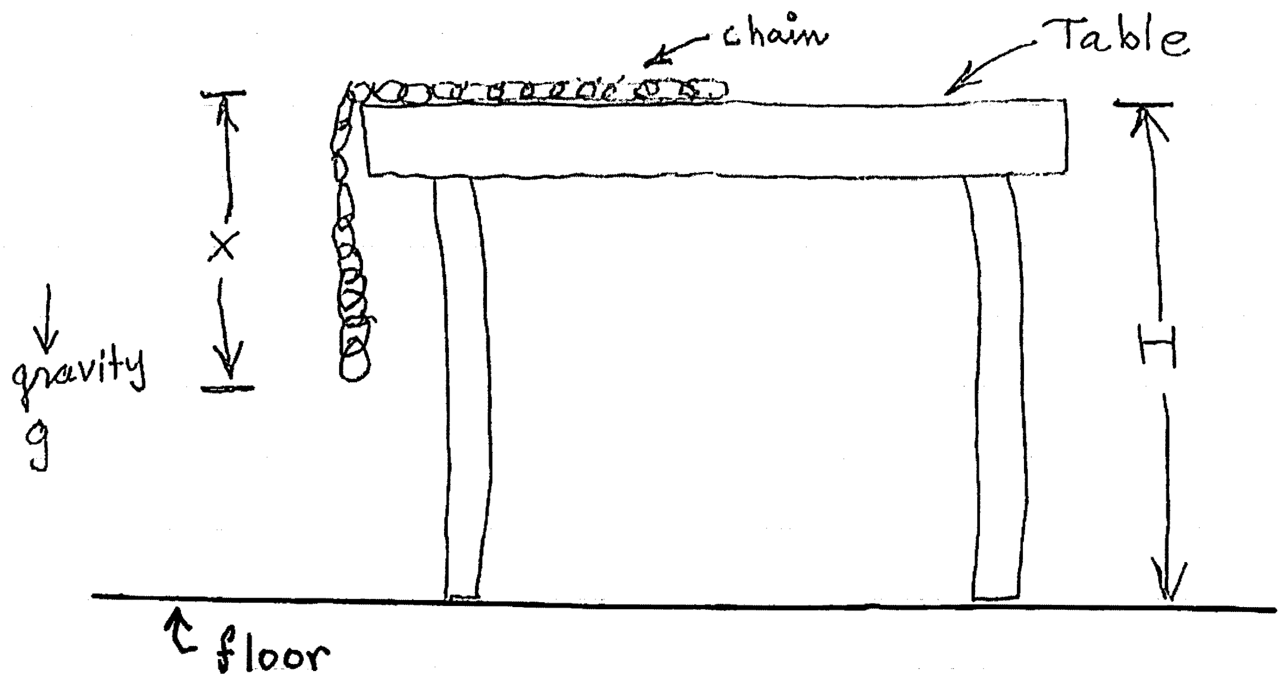
Basic Physics – Spring 2019 Written Qualifying Examination

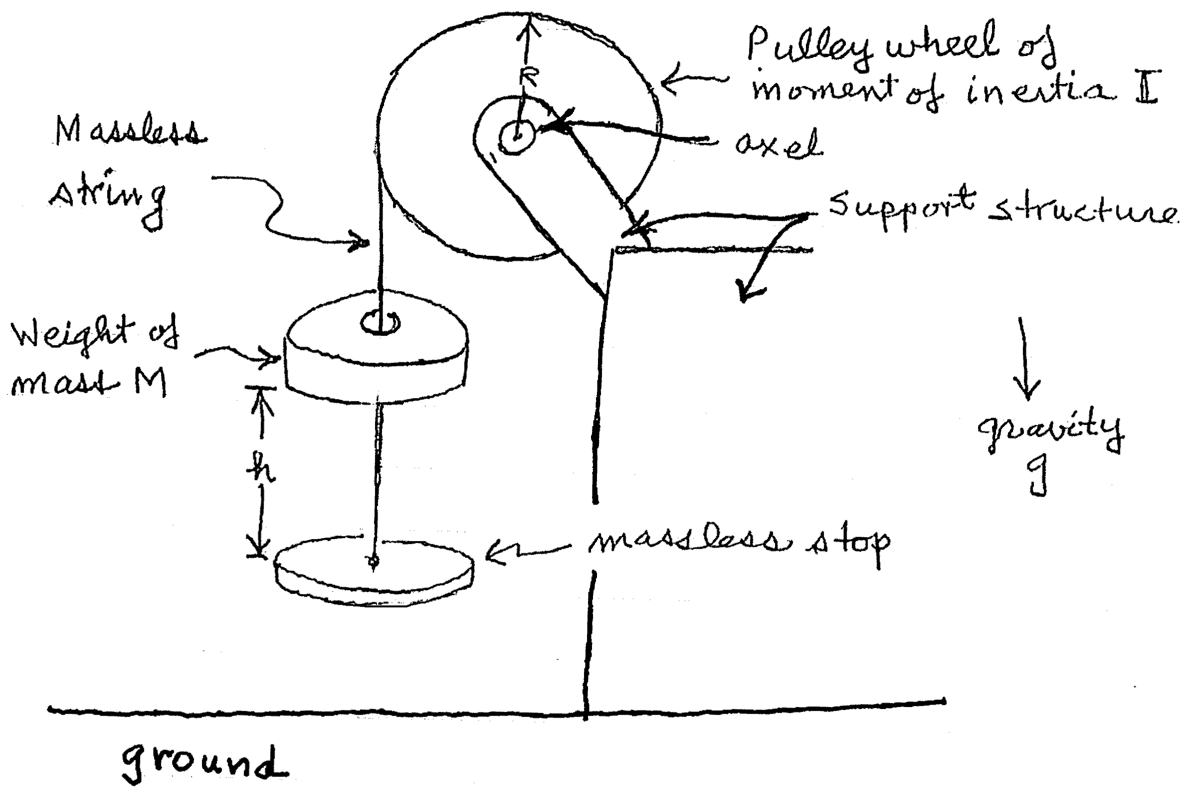
1. (Mechanics: Conservation of energy) (6 points)

As show in the figure, a chain of mass per unit length λ and total length L hangs over the edge of a stationary table. The hanging portion of the chain has length $x(t)$, while the rest of the chain (of length $L - x(t)$) lies on the frictionless surface of the tabletop. The tabletop is at a height $H < L$ above the floor. Let g denote the gravitational acceleration. For $t < 0$ the right end of the chain is held fixed and $x(t) = H/2$. At $t = 0$ the chain is released and begins to fall $(dx(t)/dt > 0)$. What is the speed V of the end of the chain at the instant just before the chain makes contact with the floor? Express your answer for V in terms of L, H and g.



2. (Mechanics: Angular momentum) (7 points)

Consider the situation shown in the diagram below. The position of the support structure is fixed to the ground, and the pulley/axle interaction is frictionless. Initially, the pulley wheel (of moment of inertia *I*) is at rest and the weight is held at the height *H* above the stop. The stop is assumed to be massless. Then the weight is released and begins to fall freely until it hits the stop.



a) (4 points) Consider the angular momentum of the composite system formed by the pulley, plus string, plus weight, plus stop. Why is <u>the total angular momentum of this composite system the same at the instant before the weight hits the stop and at the instant after the weight hits the stop, *provided* that the angular momentum is taken about the center of the pulley wheel?</u>

b) (3 points) Assuming that the collision between the weight and the stop is completely inelastic, use the underlined fact in problem 2(a) to determine the ratio (V_+/V_-) giving the decrease of the downward speed of the weight caused by the collision (V_- and V_+ denote the downward speeds of the weight at the instants just before and just after the collision, respectively).

3. (Quantum Mechanics) (7 points)

A beam of particles from $x < 0$ is normally incident on a step increase of the potential V such that $V(x) = 0$ for $x < 0$ and $V(x) = V_0 > 0$ for $x > 0$. Each particle in the beam has an energy E and a mass m and moves in the positive x- direction (i.e., the beam is normally incident on the $x = 0$ surface). Assuming $E > V_0$, use Schrödinger's equation to derive an expression for the fraction of the incident particles that are reflected.

Given information: For a point particle of mass m in the presence of a potential V, the time-dependent version of Schrödinger's equation in one dimension (coordinate x) is

$$i\hbar \frac{\partial \Psi(x,t)}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2 \Psi(x,t)}{\partial x^2} + V(x)\Psi(x,t),$$

where $\Psi(x,t)$ is the wave function and $\hbar$ is Planck's constant.

Solutions to Basic Physics Problems

Problem 1 Let (PE) = [the gravitational potential energy of the chain relative to the table top]

Let $x(t)$ = [the length of the hanging portion of the chain]

$$(PE) = -\int_0^{x(t)} \lambda g\, y\, dy = -\tfrac{1}{2}\lambda g\, x^2(t)$$

$$(KE) = [\text{Kinetic energy of the chain}] = \tfrac{1}{2}\lambda L\, v^2(t)$$

From conservation of energy, when $x(t) = H$: $\frac{g}{2}\left(H^2 - \frac{H^2}{4}\right) = \frac{1}{2} L v^2$; $v = \frac{H}{2}\sqrt{\frac{3g}{L}}$

Problem 2(a) At the instant the weight hits the stop, the stop experiences a downward impulsive force that is transmitted to the pully wheel by the string and is balanced by an upward impulsive force from the support structure. Since the pulley/axel interaction is frictionless, the latter force is normal to the circular surface of the axel and hence passes through the center of the pulley. Thus the impulsive torque on the composit system due to this force is zero, and the collision conserves the angular momentum of the composit system.

Problem 2(b) $R m v_- = R m v_+ + I v_+/R$

$$v_+/v_- = 1/\left[1 + \frac{I}{mR^2}\right]$$

Problem 3 $-(\hbar^2/2m)\frac{d^2\psi(z)}{dz^2} + V(z)\psi(z) = E\psi(z)$

$z < 0$: $\psi = e^{ik_- z} + \rho e^{-ik_- z}$, $\frac{\hbar^2}{2m}k_-^2 = E$, $k_- = \frac{\sqrt{2mE}}{\hbar}$

$z > 0$: $\psi = \tau e^{ik_+ z}$, $\frac{\hbar^2}{2m}k_+^2 = E - V_0$, $k_+ = \frac{\sqrt{2m(E - V_0)}}{\hbar}$

Continuity of ψ at $z = 0$: $1 + \rho = \tau$

Continuity of $d\psi/dz$ at $z = 0$: $k_-(1 - \rho) = \tau k_+$

Solution for ρ: $\rho = (k_- - k_+)/(k_- + k_+)$

(Fraction reflected) $= |\rho|^2 = \left[\sqrt{E} - \sqrt{E - V_0}\right]^2 / \left[\sqrt{E} + \sqrt{E - V_0}\right]^2$