1. (5 points) Boolean Simplification.

Using K-maps, determine all the minimal sums for the following incomplete Boolean function.

\[ f(w, x, y, z) = \sum m(2, 3, 7, 10, 13, 15) + dc(0, 4, 8, 14). \]

Recall that \( dc(0, 4, 8, 14) \) means that the evaluation of \( f \) on the minterms \( m_0, m_4, m_8, m_{14} \) is undefined and should be chosen in such a way as to minimize the total cost.

\[ \overline{\bar{w} \bar{x} z} + \bar{w} y z + w x \bar{z} \]

2. (4 points) Boolean Algebra.

Using Boolean Algebra postulates and theorems prove that

\[ xy + x\bar{z} = (w + x + y)(\bar{x} + y + \bar{z})(w + x + \bar{y})(\bar{w} + x) \]

No credit will be given for solutions that use the truth table method.

\[ \begin{align*}
  \text{RHS} &= (w + x + y)(w + y + \bar{z}) (\bar{w} + x) (\bar{x} + y + \bar{z}) \\
  &= [w + x + y] (\bar{w} + x) (\bar{x} + y + \bar{z}) \\
  &= (w + x + y) (\bar{w} + x) (\bar{x} + y + \bar{z}) \\
  &= (w + x + y) (\bar{w} + x) (\bar{x} + y + \bar{z}) \\
  &= (w + x + y) (\bar{w} + x) (\bar{x} + y + \bar{z}) \\
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  &= (w + x + y) (\bar{w} + x) (\bar{x} + y + \bar{z}) \\
  &= (w + x + y) (\bar{w} + x) (\bar{x} + y + \bar{z}) \\
  &= \text{LHS}
\end{align*} \]
3. (6 points) Flip-Flops.
Recall the JK Master-Slave Flip-Flop pictured below.

(a) (3 points) Fill in the following function table, where $Q^+$ denotes the output $Q$ in response to the inputs.

<table>
<thead>
<tr>
<th>Inputs</th>
<th>Outputs</th>
</tr>
</thead>
<tbody>
<tr>
<td>$J$ $K$ $C$</td>
<td>$Q^+ \ \bar{Q}^+$</td>
</tr>
<tr>
<td>0 0 x</td>
<td>0 \ Q</td>
</tr>
<tr>
<td>0 1 x</td>
<td>0 \ 1</td>
</tr>
<tr>
<td>1 0 x</td>
<td>1 \ 0</td>
</tr>
<tr>
<td>1 1 x</td>
<td>0 \ Q</td>
</tr>
<tr>
<td>x x 0</td>
<td>1 \ Q</td>
</tr>
</tbody>
</table>

(b) (3 points) Assume the control signal is 1, the slave latch is in its 1-state and a logic 0 is on both the $J$ and $K$ input lines. Then the $K$ input line switches to logic 1 briefly and then back to logic 0. What happens to the slave state when the control signal returns to 0? Explain your answer.

Let $S_m,R_m$ denote the input signals on the master latch and $S_s,R_s$ denote the input signals on the slave latch. When $K$ switches to logic 1, $R_m$ switches to logic 1, while $S_m$ remains at logic 0. So the master latch gets reset to 0, (i.e. $Q_m=0$ and $\bar{Q}_m=1$). When $K$ goes back to 0, $R_m$ switches to 0 and the master latch remains in the 0 state (i.e. $Q_m=0$ and $\bar{Q}_m=1$). Therefore, when the control signal returns to 0, the state of the slave latch goes to 0 (i.e. $Q_s=0$ and $\bar{Q}_s=1$).
4. (5 points) State Diagram.

Draw the state diagram of a minimal Mealy machine having input line \( x \), in which the signals \( \{0,1\} \) are applied, and a single output line \( y \). For \( i = 1, 2, 3, 4, \ldots \) let \( x_i \) denote the \( i \)-th input values. For \( i \in \{-2, -1, 0\} \) let \( x_i := 0 \). For \( i \geq 1 \), the system is to produce an output of 1 coincident with input symbol \( x_i \) if the binary number represented by \((x_{i-3}, x_{i-2}, x_{i-1}, x_i)\) is greater than or equal to 8 and 0 otherwise (where \( x_{i-3} \) is the most significant bit and \( x_i \) is the least significant bit).

An example of input/output sequences that satisfy the conditions of the system specification is:

<table>
<thead>
<tr>
<th>( i )</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x )</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>( y )</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
</tbody>
</table>

In the example above, the system produces an output of 1 coincident with the 4-th input symbol. This occurs since the 1-st, 2-nd, 3-rd and 4-th input symbols are 1, 0, 1, 1, which represents the decimal number 11. Since 11 \( \geq 8 \), the system outputs 1.

Your state diagram should have the minimum number of states possible.