Solutions

ECE Written Qualifying Examination, Spring 2019
Digital Logic

1. (5 points) Boolean Simplification.

   Using K-maps, determine all the minimal sums for the following incomplete Boolean function.

$$f(w,x,y,z) = \sum m(2,3,7,10,13,15) + dc(0,4,8,14).$$

   Recall that $dc(0,4,8,14)$ means that the evaluation of f on the minterms m_0, m_4, m_8, m_{14} is undefined and should be chosen in such a way as to minimize the total cost.

wx \ yz	00	01	11	10
00	—	0	1	1
01	—	0	1	0
11	0	1	1	—
10	—	0	0	1

$$\bar{x}\bar{z} + \bar{w}yz + wxz$$

2. (4 points) Boolean Algebra.

   Using Boolean Algebra postulates and theorems prove that

$$xy + x\bar{z} = (w+x+y)(\bar{x}+y+\bar{z})(w+x+\bar{y})(\bar{w}+x)$$

   No credit will be given for solutions that use the truth table method.

$RHS = (w+x+y)(w+x+\bar{y})(\bar{w}+x)(\bar{x}+y+\bar{z})$ — commutative

$= [w+x+(y\cdot\bar{y})](\bar{w}+x)(\bar{x}+y+\bar{z})$ — distributive

$= (w+x+0)(\bar{w}+x)(\bar{x}+y+\bar{z})$ — complement

$= (w+x)(\bar{w}+x)(\bar{x}+y+\bar{z})$ — identity

$= [x+(w\cdot\bar{w})](\bar{x}+y+\bar{z})$ — distributive

$= (x+0)(\bar{x}+y+\bar{z})$ — complement

$= x(\bar{x}+y+\bar{z})$ — identity

$= x\bar{x} + xy + x\bar{z}$ — distributive

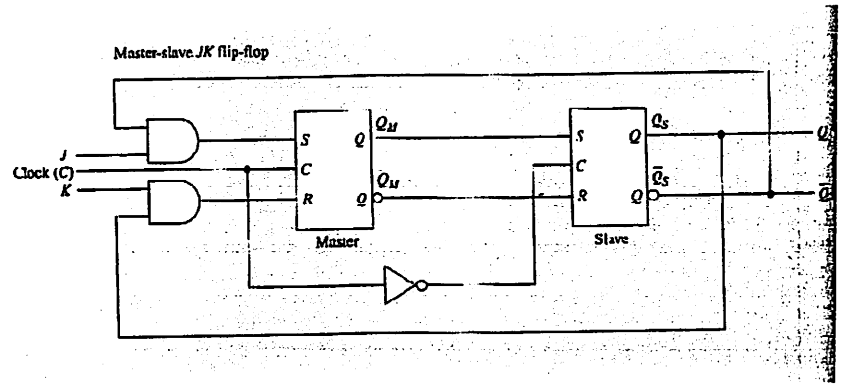
$= 0 + xy + x\bar{z}$ — complement

$= xy + x\bar{z}$ — identity

$= LHS$

3. (6 points) Flip-Flops.

Recall the JK Master-Slave Flip-Flop pictured below.



(a) (3 points) Fill in the following function table, where Q^+ denotes the output Q in response to the inputs.

Inputs			Outputs	
J	K	C	Q^+	$\overline{Q}^+$
0	0	⎍	Q	$\overline{Q}$
0	1	⎍	0	1
1	0	⎍	1	0
1	1	⎍	Q	Q
X	X	0	Q	$\overline{Q}$

(b) (3 points) Assume the control signal is 1, the slave latch is in its 1-state and a logic 0 is on both the J and K input lines. Then the K input line switches to logic 1 briefly and then back to logic 0. What happens to the slave state when the control signal returns to 0? Explain your answer.

Let S_M, R_M denote the input signals on the master latch and S_S, R_S denote the input signals on the slave latch. When K switches to logic 1, R_M switches to logic 1, while S_M remains at logic 0. So the master latch gets reset to 0. (i.e. $Q_M = 0$ and $\overline{Q}_M = 1$). When K goes back to 0, R_M switches to 0 so the master latch remains in the 0 state (i.e. $Q_M = 0$ and $\overline{Q}_M = 1$). Therefore, when the control signal returns to 0, the state of the slave latch goes to 0 (i.e. $Q_S = 0$ and $\overline{Q}_S = 1$).

4. (5 points) State Diagram.

Draw the state diagram of a *minimal* Mealy machine having input line x, in which the signals $\{0,1\}$ are applied, and a single output line y. For $i = 1, 2, 3, 4, \ldots$ let x_i denote the i-th input values. For $i \in \{-2, -1, 0\}$ let $x_i := 0$. For $i \geq 1$, the system is to produce an output of 1 coincident with input symbol x_i if the binary number represented by $(x_{i-3}, x_{i-2}, x_{i-1}, x_i)$ is greater than or equal to 8 and 0 otherwise (where x_{i-3} is the most significant bit and x_i is the least significant bit.

An example of input/output sequences that satisfy the conditions of the system specification is:

i	1	2	3	4	5	6	7	8	9	10	11
x	1	0	1	1	1	0	1	0	1	1	0
y	0	0	0	1	0	1	1	1	0	1	0

In the example above, the system produces an output of 1 coincident with the 4-th input symbol. This occurs since the 1-st, 2-nd, 3-rd and 4-th input symbols are 1, 0, 1, 1, which represents the decimal number 11. Since $11 \geq 8$, the system outputs 1.

Your state diagram should have the minimum number of states possible.