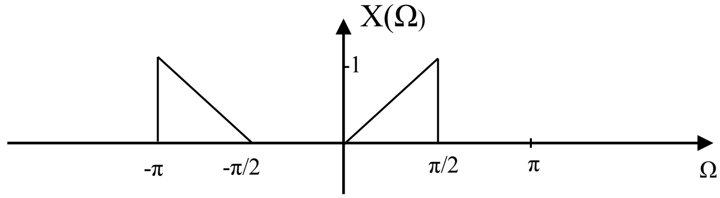
1. (4 points) The sequence x[n] has a real-valued discrete-time Fourier transform $X(\Omega)$ which is shown below for $-\pi \leq \Omega < \pi$.



Find values of Ω_0 in the range $-\pi \leq \Omega_0 < \pi$ for which $y[n] = x[n]e^{j\Omega_o n}$ is real valued. State the reasons.

2. (4 points) Given the following conditions of a continuous-time, linear and time-invariant system:
(a) The system function is rational, has only two poles at $s = -2$ and $s = 3$, and has no more than one zero.
(b) If $x(t) = 1$, then $y(t) = 0$
   (i) What is the region of convergence? Justify your answer. (2 points)
   (ii) Is the system stable? Justify your answer. (1 point)
   (iii) Is the system causal? Justify your answer. (1 point)

3. (6 points) A causal discrete-time, linear and time-invariant system has the following system function

$$H(z) = \frac{(z + 0.9)(z - 0.81)}{(z - j0.9)(z - a)}$$

You are also given that for this filter, when the input is $x[n] = (-1)^n$, the output is $y[n] = 0.1(-1)^n$.
   (i) Determine the location of the pole a. (2 points)
   (ii) Does the impulse response of this filter have finite duration? Justify your answer. (1 point)
   (iii) Is the impulse response of this filter real-valued? Justify your answer. (1 point)
   (iv) Would you classify this filter as lowpass, highpass, or bandpass? Justify your answer. (2 points)

4. (6 points) Let x[n] below be the input to an discrete-time, linear and time-invariant system whose impulse response h[n] is given below. What is the output y[n]?

$$x[n] = \sum_{k=-\infty}^{\infty} \delta[n - 2k] \qquad\qquad h[n] = \frac{\sin\left(\frac{\pi n}{2}\right)}{\pi n}$$

Answers:

Part 1. $\Omega_o = \frac{\pi}{4}, \frac{-3\pi}{4}$

Part 2.

(i) ROC: $-2 < \sigma < 3$, given s=0 is part of the ROC.

(ii) The system is stable given the ROC includes the $j\omega$ axis.

(iii) The system is not causal. It is two-sided given the ROC is a vertical strip (not to the right of the rightmost pole and not left of the leftmost pole.).

Part 3.

(i) a=-j0.9

(ii) System is IIR since there are poles at frequencies other than 0 and ∞.

(iii) It does have a real impulse response since the poles must be complex conjugates of each other in order for $H(-1) = 0.1$. That is, the length of the vector from $\Omega = \pi$ $(i.e., z = -1)$ to the pole must be $\sqrt{1.81}$ and the angle of the vector must be opposite and equal to the vector from $\Omega = \pi$ to the pole at $z = 0.9j$.

(iv) It is a bandpass filter given it has a zero close to $\omega = 0$ and $\omega = \pi$, and it peaks at $\omega = \pi/2$.

Part 4. $X(\Omega) = \pi \sum_{k=-\infty}^{\infty} \delta(\Omega - \pi k)$ and $H(\Omega) = \begin{cases} 1 & |\Omega| < \frac{\pi}{2} \\ 0 & otherwise \end{cases}$, so that $Y(\Omega) = X(\Omega)\mathrm{H}(\Omega) = \pi \sum_{k=-\infty}^{\infty} \delta(\Omega - 2\pi k)$ which means that $y[n] = 1/2$.