ENEE680 Final Exam Due Dec. 19, 2011

1. A cylindrical rod of radius a and permeability μ is surrounded by an annular region of material ($a < r < b$) with fixed, but spatially varying, magnetization $\mathbf{M} = \hat{\mathbf{r}} M_0 \cos l\theta \, r / a$. Outside this region ($r > b$) is vacuum. Nothing depends on z, and there is no free current density.

a) (10 pts) What is the induced current density implied by the magnetization?

b) (10 pts) What equations does the magnetic scalar potential satisfy in each of the three regions?

c) (10 pts) What boundary conditions are applied to the magnetic scalar potential at: $r = 0$, $r=a$, $r=b$, and $r \to \infty$?

d) (10 pts) Solve for the magnetic scalar potential in each region and then indicate how you would calculate **B** and **H** in each region?

e) (10 pts) Sketch the magnetic (**B**) field lines in the $r-\theta$ plane for $0 < \theta < 2\pi / l$.

2. A time dependent free surface current flows in windings on the cylindrical surface at r=b of a long solenoid. The surface current density is $\mathbf{J_s} = \mathbf{e}_\theta J_0 e^{\gamma t}$ [A/m].

A) Find the magnetic flux density inside and outside the solenoid including the effect of Maxwell's displacement current.
1) (10 pts) To do this you must determine which field components of **E** and **B** are nonzero.
2) (10 pts) Derive a differential equation for $B_z(r,t) = \hat{B}(r) e^{\gamma t}$.
3) (10 pts) What boundary conditions apply at r=0, r=b and $r \to \infty$.
4) (10 points) Show that $\hat{B}(r)$ for $r < b$ is given by $\hat{B}(r) = kb\mu_0 J_0 I_0(kr) K_0'(kb)$, where $k = \gamma / c$. (Caution J_0 is the surface current density, not a Bessel function. K_o and I_0 are modified Bessel functions.) Here prime means derivative with respect to argument. Find a similar expression for r>b.
5) (10 pts) Plot schematically $\hat{B}(r)$ the limits in which γ is small and large. Verify that you recover reasonable results. Explain the physics behind these two limits.

B) Now, in the limit that γ is small, (effectively this means neglect the displacement current) add a conducting cylinder of radius *a*<*b* in the center of the solenoid to the problem.
1) (10 pts) Rederive the differential equation for $B_z(r,t) = \hat{B}(r) e^{\gamma t}$ including the effect of the conductivity on the cylinder.
2) (10 pts) What boundary conditions are satisfied at r=a?
3) (10 pts) Find the fields in the cylinder and in the region between the cylinder and the surface current. By now you must be a Bessel function master.

Useful formulas: $I_0'(x)K_0(x) - I_0(x)K_0'(x) = 1/x$ Here prime means derivative with respect to argument.

Small x: $I_0(x) \simeq 1$, $K_0(x) \simeq -\ln x$ Large x: $I_0(x) \sim (2\pi x)^{-1/2} e^{-x}$, $K_0(x) \sim (\pi/2x)^{1/2} e^{x}$