

Solutions

ENEE/CMSC/MATH 456: Cryptography
CRHF Class Exercise 3/25/19

Let (Gen, H) be a collision-resistant hash function and let F be a PRF. For each of the following, state whether \hat{H} is necessarily collision resistant. Justify your answer.

1. $\hat{H}^s(x_1 || x_2) = H^s(x_1 \oplus F_s(x_2))$ No.

Attack: Choose arbitrary x_1, x_2, x'_2

set $x'_1 = x_1 \oplus F_s(x_2) \oplus F_s(x'_2)$ [note adv can find such x'_1 since s is public]

output $x_1 || x_2, x'_1 || x'_2$

$$\hat{H}^s(x_1 || x_2) = H^s(x_1 \oplus F_s(x_2))$$

$$\hat{H}^s(x'_1 || x'_2) = H^s(x'_1 \oplus F_s(x'_2)) = H^s(x_1 \oplus F_s(x_2) \oplus F_s(x'_2) \oplus F_s(x'_2)) = H^s(x_1 \oplus F_s(x_2)).$$

2. $\hat{H}^s(x_1 || x_2) = H^s(H^s(x_1) || x_2)$ Yes.

Assume towards contradiction that an attacker finds $x_1 || x_2 \neq x'_1 || x'_2$ such that $\hat{H}^s(x_1 || x_2) = \hat{H}^s(x'_1 || x'_2)$.

$$\text{Then } H^s(H^s(x_1) || x_2) = H^s(H^s(x'_1) || x'_2) = y$$

$$\text{Let } y_2 = H^s(x_1) || x_2$$

$$y'_2 = H^s(x'_1) || x'_2$$

Case 1: $y'_2 \neq y_2$. Then we have found a coll. on the outer instantiation of H^s since $H^s(y_2) = H^s(y'_2) = y$.

Case 2: $y_2 = y'_2$. Then $x_2 = x'_2$. Since $x_1 || x_2 \neq x'_1 || x'_2$ it means that $x_1 \neq x'_1$. But this means we have found a coll. on the inner instantiation of H^s since $H^s(x_1) = H^s(x'_1)$ in order for $y_2 = y'_2$.