

Let G be a pseudorandom generator where $|G(s)| = |s| + 1$

1. Define $G'(s) = G(s||\bar{s})$, where \bar{s} is the bit-wise negation of s . Is G' necessarily a pseudorandom generator? No.

Let G^* be a PRG from inputs of length n to length $2n+1$
 Define G in terms of G^* as follows: $G(s=s_1||s_2) := G^*(s_1 \oplus s_2)$
 G is a PRG from n to $n+1$. G is secure b/c $s_1 \oplus s_2$ is unif. dist.
 Note $G'(s) = G(s||\bar{s}) = G^*(s \oplus \bar{s}) = G^*(1^n) = \text{constant}$.

Distinguisher for G' :
 $D(w)$:
 if $w = G^*(1^n)$ output 1
 else output 0

Need to show $|\Pr[D(r)=1] - \Pr[D(G'(s))=1]|$ is high.

2. Define $G'(s) = G(s)||G(\bar{s})$, where \bar{s} is the bit-wise negation of s . Is G' necessarily a pseudorandom generator? No.

Let G^* be a PRG from inputs of length n to $n+2$.
 Define G in terms of G^* as follows.

$G(s=s_1||s')$: [where s_1 is a single bit]

if $s_1 = 0$, output $G^*(s')$	}	G is PRG from n to $n+1$. G is secure b/c s', \bar{s}' unif. dist.
if $s_1 = 1$, output $G^*(\bar{s}')$		

Note $G'(s, s') = G(s, s') || G(\bar{s}, \bar{s}') = G^*(s) || G^*(s)$

Distinguisher checks if 1st and 2nd half of w are the same. Need to show $|\Pr[D(r)=1] - \Pr[D(G'(s))=1]|$ is high

3. Define $G'(s) = G(s)_1 || G(G(s)_2, \dots, G(s)_{|s|+1})$, where $G(s)_i$ denotes the i -th output bit of $G(s)$. Is G' necessarily a pseudorandom generator? Yes.

Use a hybrid argument. Consider 3 distributions H_0, H_1, H_2
 In order to prove $G'(s)$ is a PRG, need to show distinguisher cannot dist. H_0, H_2

$H_0: G(s)_1 || G(G(s)_2, \dots, G(s)_{|s|+1})$ where $s \leftarrow_R \{0,1\}^n$

$H_1: r_1 || G(r_2, \dots, r_{|s|+1})$ where $r \leftarrow_R \{0,1\}^{n+1}$

Indistinguishable due to security of PRG G .

$H_2: r_1 || r'_1, \dots, r'_{n+1}$ where $r_1 \leftarrow_R \{0,1\}, r' \leftarrow_R \{0,1\}^{n+1}$

Indistinguishable due to security of PRG G .

Note G' has stretch 2. Takes inputs of length n , produces outputs of length $n+2$.