

Digital Logic Design

ENEE 244-010x

Lecture 10

Announcements

- HW5 up on course webpage. Due on **Wednesday**, 10/21 in class.

Agenda

- Last time
 - Using 3,4 variable K-Maps to find minimal expressions (4.5)
- This time
 - Minimal expressions for incomplete Boolean functions (4.6)
 - 5 and 6 variable K-Maps (4.7)
 - Quine-McCluskey method (4.8)

Minimal Expressions of Incomplete Boolean Functions

- Recall an incomplete Boolean function has a truth table which contains dashed functional entries indicating don't-care conditions.
- Idea: Can replace don't-care entries with either 0s or 1s in order to form the largest possible subcubes.

Example

$$f(w, x, y, z) = \sum m(0,1,2,5,8,15) + dc(6,7,10)$$

		<i>yz</i>			
		00	01	11	10
<i>wx</i>	00	0	1	3	2
	01	4	5	7	6
	11	12	13	15	14
	10	8	9	11	10

Example

$$f(w, x, y, z) = \sum m(0,1,2,5,8,15) + dc(6,7,10)$$

		<i>yz</i>			
		00	01	11	10
<i>wx</i>	00	1	1	0	1
	01	0	1	--	--
	11	0	0	1	0
	10	1	0	0	--

Example

Step 1: Find prime implicants (pretend don't care cells set to 1)

		yz			
		00	01	11	10
wx	00	1	1	0	1
	01	0	1	--	--
	11	0	0	1	0
	10	1	0	0	--

Example

Step 2: Find essential prime implicants (discount don't care cells)

		yz			
		00	01	11	10
wx	00	1	1	0	1
	01	0	1	--	--
	11	0	0	1	0
	10	1	0	0	--

Essential prime implicants: $\bar{x}\bar{z}, xyz$

Example

Step 3: Add prime implicants to cover all 1-cells (discount don't care cells)

		yz			
		00	01	11	10
wx	00	1	1	0	1
	01	0	1	--	--
	11	0	0	1	0
	10	1	0	0	--

Essential prime implicants: $\bar{x}\bar{z}$, xyz

Add: $\bar{w}\bar{y}z$

Example

Step 3: Add prime implicants to cover all 1-cells (discount don't care cells)

		<i>yz</i>			
		00	01	11	10
<i>wx</i>	00	1	1	0	1
	01	0	1	--	--
	11	0	0	1	0
	10	1	0	0	--

Final minimal DNF: $\bar{x}\bar{z} + xyz + \bar{w}\bar{y}z$

Five and Six Variable K-Maps

Five Variable K-Maps

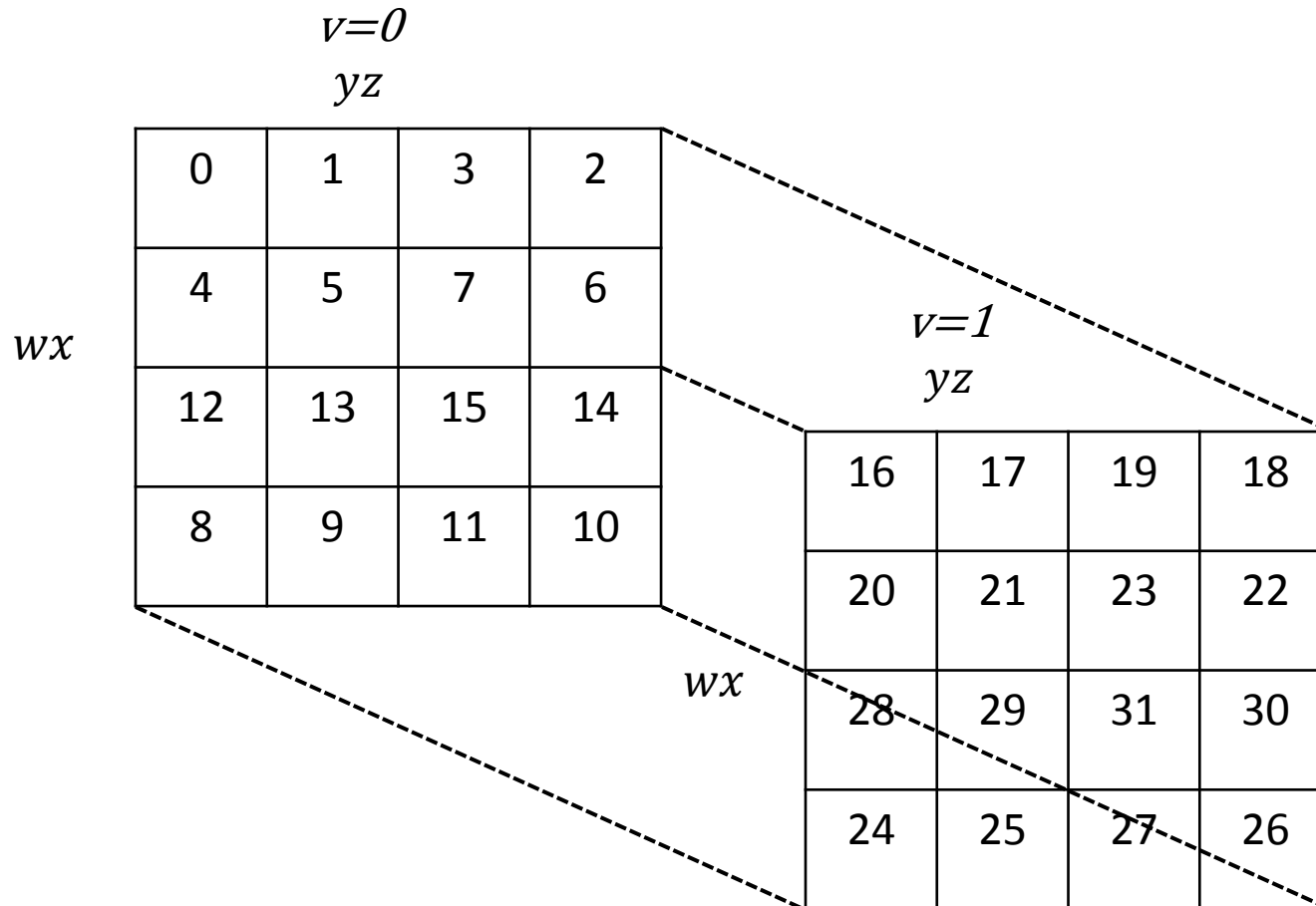
- We can visualize five-variable map in two different ways:

Five Variable K-Maps

		<i>xyz</i>							
		000	001	011	010	110	111	101	100
<i>vw</i>	00	0	1	3	2	6	7	5	4
	01	8	9	11	10	14	15	13	12
	11	24	25	27	26	30	31	29	28
	10	16	17	19	18	22	23	21	20

Subcubes: Two subcubes are possible about the mirror-image line. If there are two rectangular groupings of the same $2^a \times 2^b$ dimensions on both halves and the two groupings are the mirror image of each other.

Five Variable K-Maps



Subcubes: If each layer contains a $2^a \times 2^b$ subcube such that they can be viewed as being directly above and below each other, then the two subcubes collectively form a single subcube consisting of 2^{a+b+1} cells.

Example

$$f(v, w, x, y, z) = \sum_{v=0}^{m(1,5,9,11,13,20,21,26,27,28,29,30,31)}$$

wx

0	1	0	0
0	1	0	0
0	1	0	0
0	1	1	0

wx

v=1
yz

0	0	0	0
1	1	0	0
1	1	1	1
0	0	1	1

Example

Step 1: Find all Prime Implicants.

$v=0$
 yz

wx

0	1	0	0
0	1	0	0
0	1	0	0
0	1	1	0

wx

Detailed description: A 4x4 Karnaugh map for v=0. The vertical axis is labeled 'wx' and the horizontal axis is labeled 'yz'. The map contains 1s in the following cells: (0,1), (1,1), (2,1), (3,1), and (3,2). Three prime implicants are circled: a blue vertical rectangle covering (0,1), (1,1), and (2,1); a red vertical rectangle covering (1,1), (2,1), and (3,1); and a yellow horizontal rectangle covering (3,1) and (3,2).

$v=1$
 yz

0	0	0	0
1	1	0	0
1	1	1	1
0	0	1	1

Detailed description: A 4x4 Karnaugh map for v=1. The vertical axis is labeled 'yz' and the horizontal axis is labeled 'wx'. The map contains 1s in the following cells: (1,0), (1,1), (2,0), (2,1), (2,2), (2,3), (3,2), and (3,3). Four prime implicants are circled: a green horizontal rectangle covering (1,0), (1,1), (2,0), and (2,1); a red vertical rectangle covering (1,1) and (2,1); a yellow circle covering (3,2); and a purple horizontal rectangle covering (2,2), (2,3), (3,2), and (3,3).

Example

Step 2: Find all Essential Prime Implicants.

$v=0$
 yz

wx	00	0	1	0	0
	01	0	1	0	0
	11	0	1	0	0
	10	0	1	1	0

wx

$v=1$
 yz

	0	0	0	0
	1	1	0	0
	1	1	1	1
	0	0	1	1

Essential Prime Implicants:

$$\bar{v} \bar{y} z$$

$$v x \bar{y}$$

$$v w y$$

Example

Step 2: Find all Essential Prime Implicants.

$v=0$
 yz

wx

0	1	0	0
0	1	0	0
0	1	0	0
0	1	1	0

$v=1$
 yz

0	0	0	0
1	1	0	0
1	1	1	1
0	0	1	1

Essential Prime Implicants:

$\bar{v} \bar{y} z$
 $v x \bar{y}$
 $v w y$

wx

Example

Step 2: Find all Essential Prime Implicants.

$v=0$
 yz

	0	1	0	0
	0	1	0	0
	0	1	0	0
wx	0	1	1	0

wx

$v=1$
 yz

	0	0	0	0
	1	1	0	0
	1	1	1	1
	0	0	1	1

Final minimal DNF:
 $\bar{v} \bar{y} z + vx \bar{y} + vwy + w \bar{x} yz$

Six Variable K-Maps

- We can visualize a six-variable map in two different ways:

Six Variable K-Maps

		<i>xyz</i>							
		000	001	011	010	110	111	101	100
<i>uvw</i>	000	0	1	3	2	6	7	5	4
	001	8	9	11	10	14	15	13	12
	011	24	25	27	26	30	31	29	28
	010	16	17	19	18	22	23	21	20
	110	48	49	51	50	54	55	53	52
	111	56	57	59	58	62	63	61	60
	101	40	41	43	42	46	47	45	44
	100	32	33	35	34	38	39	37	36

Subcubes: If each quadrant has a rectangular grouping of dimensions $2^a \times 2^b$ and each grouping is a mirror image of the other about both the horizontal and vertical mirror-image lines.

Six Variable K-Maps

$uv=00$
 yz

	0	1	3	2
	4	5	7	6
wx	12	13	15	14
	8	9	11	10

$uv=01$
 yz

	16	17	19	18
	20	21	23	22
wx	28	29	31	30
	24	25	27	26

$uv=11$
 yz

	48	49	51	50
	52	53	55	54
wx	60	61	63	62
	56	57	59	58

$uv=10$
 yz

	32	33	35	34
	36	37	39	38
wx	44	45	47	46
	40	41	43	42

Subcubes: Subcubes occurring in corresponding positions in consecutive layers collectively form a single subcube.

Quine-McCluskey method

Tabular Representations

		yz			
		00	01	11	10
wx	00	0	0	0	1
	01	1	1	1	1
	11	0	1	1	1
	10	0	1	0	0

$\bar{w}x$
 01 --

$\bar{w}y\bar{z}$
 0 - 10

$w\bar{y}z$
 1 - 01

xy
 -11 -

$$f(w, x, y, z) = xy + \bar{w}y\bar{z} + w\bar{y}z + \bar{w}x$$

Prime Implicants

- $f(w, x, y, z) = xy + \bar{w}y\bar{z} + w\bar{y}z + \bar{w}x$
- Each product term is an implicant.
- Prime implicant: A product term that cannot have any of its literals removed and still imply the function.

Prime Implicants

		<i>yz</i>			
		00	01	11	10
<i>x</i>	0	0	0	0	1
	1	1	1	1	1

1---

-10

Prime Implicants

Minterm	X	Y	Z	F
0	0	0	0	0
1	0	0	1	0
2	0	1	0	1
3	0	1	1	0
4	1	0	0	1
5	1	0	1	1
6	1	1	0	1
7	1	1	1	1

		yz			
		00	01	11	10
x	0	0	0	0	1
	1	1	1	1	1

-10

1 - -

$$f(x, y, z) = y\bar{z} + x$$

Finding Prime Implicants

Step 1					Step 2				Step 3			
2	0	1	0	✓	(2,6)	—	1	0	(4,5,6,7)	1	—	—
4	1	0	0	✓	(4,5)	1	0	—	(4,6,5,7)	1	—	—
5	1	0	1	✓	(4,6)	1	—	0				
6	1	1	0	✓	(5,7)	1	—	1				
7	1	1	1	✓	(6,7)	1	1	—				

All unchecked entries are Prime Implicants

$$-10 \quad y\bar{z}$$

$$1- - \quad x$$

Prime Implicants

Minterm	X	Y	Z	F
0	0	0	0	0
1	0	0	1	0
2	0	1	0	1
3	0	1	1	0
4	1	0	0	1
5	1	0	1	1
6	1	1	0	1
7	1	1	1	1

		yz			
		00	01	11	10
x	0	0	0	0	1
	1	1	1	1	1

-10

1 - -

$$f(x, y, z) = y\bar{z} + x$$