Digital Logic Design
ENEE 244-010x

Lecture 11

Partly based on video from:
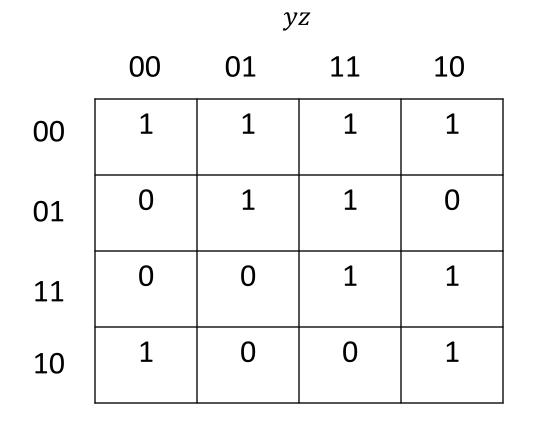https://www.youtube.com/watch?v=pQ3MfzqGlrc

Announcements

- HW5 due on Wednesday, 10/21

Agenda

- Last time:
  - Minimal expressions for incomplete Boolean functions (4.6)
  - 5 and 6 variable K-Maps (4.7)
  - Quine-McCluskey method (4.8)

- This time:
  - Quine-McCluskey method (4.8) (another example)
  - Petrick's method of determining irredundant expressions (4.9)
  - Prime Implicant Table Reductions (4.10)
  - The Multiple Output Simplification Problem (4.12)
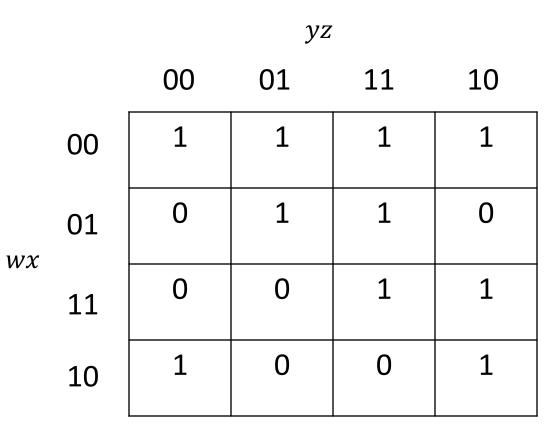
Essential Prime Implicants

yz

	00	01	11	10
00	1	1	1	1
01	0	1	1	0
11	0	0	1	1
10	1	0	0	1

wx

Find the Essential Prime Implicants using Quine-McClusky

Essential Prime Implicants

yz

	00	01	11	10
00	1	1	1	1
01	0	1	1	0
11	0	0	1	1
10	1	0	0	1

wx

minterms

0	0 0 0 0
1	0 0 0 1
2	0 0 1 0
8	1 0 0 0
3	0 0 1 1
5	0 1 0 1
10	1 0 1 0
7	0 1 1 1
14	1 1 1 0
15	1 1 1 1

Finding Prime Implicants

0	0 0 0 0	✔
1	0 0 0 1	✔
2	0 0 1 0	✔
8	1 0 0 0	✔
3	0 0 1 1	✔
5	0 1 0 1	✔
10	1 0 1 0	✔
7	0 1 1 1	✔
14	1 1 1 0	✔
15	1 1 1 1	✔

(0,1)	0	0	0	−	✔
(0,2)	0	0	−	0	✔
(0,8)	−	0	0	0	✔
(1,3)	0	0	−	1	✔
(1,5)	0	−	0	1	✔
(2,3)	0	0	1	−	✔
(2,10)	−	0	1	0	✔
(8,10)	1	0	−	0	✔
(3,7)	0	−	1	1	✔
(5,7)	0	1	−	1	✔
(10,14)	1	−	1	0
(7,15)	−	1	1	1
(14,15)	1	1	1	−

(0,1,2,3)	0	0	−	−
(0,2,1,3)	0	0	−	−
(0,2,8,10)	−	0	−	0
(0,8,2,10)	−	0	−	0
(1,3,5,7)	0	−	−	1
(1,5,3,7)	0	−	−	1

> ### 6 Prime Implicants
>
> $1 − 10$ $00 − −$
> $−111$ $−1 − 1$
> $111 −$ $0 − −1$

Find Essential Prime Implicants

Prime Implicant	Covered Minterms	Minterms									
		0	1	2	3	5	7	8	10	14	15
1—10	10,14								X	X	
—111	7,15						X				X
111—	14,15									X	X
00— —	0,1,2,3	X	X	X	X						
—0—0	0,2,8,10	X		X				X	X		
0— —1	1,3,5,7		X		X	X	X				

3 Prime Implicants

$$0--1$$

yz

	00	01	11	10
00	1	1	1	1
01	0	1	1	0
11	0	0	1	1
10	1	0	0	1

wx

$$111-$$

$$-0-0$$

3 Prime Implicants

yz

$0- -1$

wx	00	01	11	10
00	1	1	1	1
01	0	1	1	0
11	0	0	1	1
10	1	0	0	1

$111-$

$-0-0$

$$f(w, x, y, z) = \overline{w}z + \overline{x}\,\overline{z} + wxy$$

The Final Step in Expression Minimization

Petrick's Method of Determining Irredundant Expressions

		m_4	m_5	m_9	m_{11}	m_{12}	m_{14}	m_{15}	m_{27}	m_{30}
A	$\overline{x}\,\overline{y}z$			X						
B	$w\,\overline{x}z$			X	X				X	
C	wyz				X			X	X	
D	wxy						X	X		X
E	vwy								X	X
F	$\overline{v}\,\overline{w}\,\overline{y}z$		X							
G	$\overline{v}\,\overline{w}\,x\overline{y}$	X	X							
H	$\overline{v}x\overline{y}\,\overline{z}$	X				X				
I	$\overline{v}wx\overline{z}$					X	X			

The covering problem: Determine a subset of prime implicants that covers the table. A minimal cover is an irredundant cover that corresponds to a minimal sum of the function.

Petrick's Method of Determining Irredundant Expressions

		m_4	m_5	m_9	m_{11}	m_{12}	m_{14}	m_{15}	m_{27}	m_{30}
A	$\bar{x}\,\bar{y}z$			X						
B	$w\,\bar{x}z$			X	X				X	
C	wyz				X			X	X	
D	wxy						X	X		X
E	vwy								X	X
F	$\bar{v}\,\bar{w}\,\bar{y}z$		X							
G	$\bar{v}\,\bar{w}\,x\bar{y}$	X	X							
H	$\bar{v}x\bar{y}\,\bar{z}$	X				X				
I	$\bar{v}wx\bar{z}$					X	X			

p-expression: (G+H)(F+G)(A+B)(B+C)(H+I)(D+I)(C+D)(B+C+E)(D+E)
The p-expression equals 1 iff a sufficient subset of prime implicants is selected.

P-expressions

- If a p-expression is manipulated into its sum-of-products form using the distributive law, duplicate literals deleted in each resulting product term and subsuming product temrms deleted, then each remaining product term represents an irredundant cover of the prime implicant table.

- Since all subsuming product terms have been deleted, the resulting product terms must each describe an irredundant cover.

- The irredundant DNF is obtained by summing the prime implicants indicated by the variables in a product term.

Simplifying p-expressions

$p =$ (G+H)(F+G)(A+B)(B+C)(H+I)(D+I)(C+D)(B+C+E)(D+E)

$=$ (G+H)(F+G)(A+B)(B+C)(H+I)(D+I)(C+D)(D+E)

$=$ (G+HF)(B+AC)(I + DH)(D+CE)

$=$ (BG + ACG + BFH + ACFH)(DI + CEI + DH + CDEH)

$=$ (BG + ACG + BFH + ACFH)(DI + CEI + DH)

$=$ BDGI + BCEGI + BDGH + ACDGI +ACEGI + ACDGH + BDFHI + BCEFHI + BDFH + ACDFHI + ACEFHI + ACDFH

$=$ BDGI + BCEGI + BDGH + ACDGI +ACEGI + ACDGH + BCEFHI + BDFH + ACEFHI + ACDFH

Finding Minimal Sums

BDGI + BCEGI + BDGH + ACDGI +ACEGI + ACDGH + BCEFHI + BDFH + ACEFHI + ACDFH

- There are 10 irredundant expressions
- Evaluate each one by the cost criteria to find the minimal sum.
- Minimal DNFs correspond to the first, third and eighth terms.

Column and Row Reductions

Example

		m_4	m_5	m_9	m_{11}	m_{12}	m_{14}	m_{15}	m_{27}	m_{30}
A	$\overline{x}\,\overline{y}z$			X						
B	$w\,\overline{x}z$			X	X				X	
C	wyz				X			X	X	
D	wxy						X	X		X
E	vwy								X	X
F	$\overline{v}\,\overline{w}\,\overline{y}z$		X							
G	$\overline{v}\,\overline{w}\,x\overline{y}$	X	X							
H	$\overline{v}x\overline{y}\,\overline{z}$	X				X				
I	$\overline{v}wx\overline{z}$					X	X			

Example

Cost is 1 plus number of literals in the term. **If there is only one literal, cost is 1.

		m_4	m_5	m_9	m_{11}	m_{12}	m_{14}	m_{15}	m_{27}	m_{30}	Cost
A	$\bar{x}\,\bar{y}z$			X							4
B	$w\,\bar{x}z$			X	X				X		4
C	wyz				X			X	X		4
D	wxy						X	X		X	4
E	vwy								X	X	4
F	$\bar{v}\,\bar{w}\,\bar{y}z$		X								5
G	$\bar{v}\,\bar{w}\,x\bar{y}$	X	X								5
H	$\bar{v}x\bar{y}\,\bar{z}$	X				X					5
I	$\bar{v}wx\bar{z}$					X	X				5

Example

	m_4	m_5	m_9	m_{11}	m_{12}	m_{14}	m_{15}	m_{27}	m_{30}	Cost
A			X							4
B			X	X				X		4
C				X			X	X		4
D						X	X		X	4
E								X	X	4
F		X								5
G	X	X								5
H	X				X					5
I					X	X				5

Example

	m_4	m_5	m_9	m_{11}	m_{12}	m_{14}	m_{15}	m_{27}	m_{30}	Cost
A			X							4
B			X	X				X		4
C				X			X	X		4
D						X	X		X	4
E								X	X	4
F		X								5
G	X	X								5
H	X				X					5
I					X	X				5

Example

	m_4	m_5	m_9	m_{11}	m_{12}	m_{14}	m_{15}	m_{30}	Cost
A			X						4
B			X	X					4
C				X			X		4
D						X	X	X	4
E								X	4
F		X							5
G	X	X							5
H	X				X				5
I					X	X			5

If equal columns exist, can delete either one of them.

Example

	m_4	m_5	m_9	m_{11}	m_{12}	m_{14}	m_{15}	m_{30}	Cost
A			X						4
B			X	X					4
C				X			X		4
D						X	X	X	4
E								X	4
F		X							5
G	X	X							5
H	X				X				5
I					X	X			5

Which rows dominate E and F?

Example

	m_4	m_5	m_9	m_{11}	m_{12}	m_{14}	m_{15}	m_{30}	Cost
B			X	X					4
C				X			X		4
D						X	X	X	4
G	X	X							5
H	X				X				5
I					X	X			5

If equal rows exist, can delete the one with higher cost. If both have the same cost, can delete either one.

Example

	m_4	m_5	m_9	m_{11}	m_{12}	m_{14}	m_{15}	m_{30}	Cost
**B			X	X					4
C				X			X		4
**D						X	X	X	4
**G	X	X							5
H	X				X				5
I					X	X			5

Example

	m_{12}	Cost
H	X	5
I	X	5

Can select either H or I since they both have the same cost.

Final minimal cover is either:

B,D,G,H

B,D,G,I

Note: Unlike Petrick's method, not all minimal covers are necessarily obtained.

Cyclic Table

- If at some point, there are no more rows or columns that can be deleted using row/column reduction, this is called a cyclic table.

- At this point, must run Petrick's method on the reduced table to find the minimal sum.

Summary: Prime Implicant Selection Procedure

1.   Find all essential prime implicants. Rule a line through the essential rows and all columns which have an X in an essential row.

2.   Rule a line through all dominating columns and dominated rows, keeping in mind the cost restriction for deleting rows.

3.   Check to see if any unruled column has a single X. If there are no such columns, then the table is cyclic. If there are some columns with a single X, place a double asterisk next to the rows in which these X's appear. These are called secondary essential rows. Rule a line through each secondary essential row and each column in which an X appears in a secondary essential row.

4.   If all columns are ruled out, then the minimal sum is given by the sum of all the prime implicants which are associated with rows that have asterisks next to them. If all columns are not ruled out, then repeat Steps 2 and 3 until either there are no columns to be ruled or a cyclic table results.

5.   If a cyclic table results, then Petrick's method is applied to the cyclic table and a minimal cover is obtained for it. The sum of all prime implicants that are marked with asterisks plus the prime implicants for the minimal cover of the cyclic table as determined by Petrick's method is a minimal sum.