Class Exercise

9/14/15

Axioms and Theorems:

P1. Closure

For all $x, y \in B, x + y \in B, x \cdot y \in B$

P2. Identity

There exist identity elements in B, denoted 0,1 relative to (+) and (·), respectively.

For all $x \in B, 0 + x = x + 0 = x, 1 \cdot x = x \cdot 1 = x$.

P3. Commutativity

The operations (+), (·) are commutative

For all $x, y \in B\ x + y = y + x,\ \ x \cdot y = y \cdot x$

P4. Distributivity

Each operation (+), (·) is distributive over the other.

For all $x, y, z \in B$:

$$x + (y \cdot z) = (x + y) \cdot (x + z)\ [x\ OR\ (y\ AND\ z)]$$

$$x \cdot (y + z) = (x \cdot y) + (x \cdot z)\ [x\ AND\ (y\ OR\ z)]$$

P5. Complement

For every element $x \in B$ there exists an element $\overline{x} \in B$called the complement of x such that:

$$x + \overline{x} = 1$$

$$x \cdot \overline{x} = 0$$

P6. Non-triviality

There exist at least two elements $x, y \in B$ such that $x \neq y$.

Theorem 3.2: Null elements:

For each element x in a Boolean algebra:

$$x + 1 = 1$$

$$x \cdot 0 = 0.$$

Theorem 3.4: The Idempotent law

For each element x in a Boolean algebra:

$$x + x = x$$

$$x \cdot x = x$$

Theorem 3.5: The Involution Law

For every x in a Boolean algebra, $\overline{(\overline{x})} = x$.

Theorem 3.6: The absorption law

For each pair of elements x, y in a Boolean algebra:

$$x + x \cdot y = x$$

$$x \cdot (x + y) = x$$

Theorem 3.7: DeMorgan's Law

For each pair of elements x, y in a Boolean algebra:

$$\overline{(x + y)} = \overline{x} \cdot \overline{y}$$

$$\overline{(x \cdot y)} = \overline{x} + \overline{y}$$

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1. Prove $x + \overline{x}y = x + y$

2. Prove $\overline{x}\,\overline{y} + xy + \overline{x}y = \overline{x} + y$

3. Prove that no Boolean algebra has exactly three distinct elements.