Class Exercise

9/16/15

Shannon's Expansion Theorem:
Theorem 3.11

(a) $f(x_1, \dots, x_i, \dots, x_n) =$
$$x_i \cdot f(x_1, \dots, 1, \dots, x_n) + \overline{x}_i \cdot f(x_1, \dots, 0, \dots, x_n)$$

(b) $f(x_1, \dots, x_i, \dots, x_n) =$
$$(\overline{x}_i + f(x_1, \dots, 1, \dots, x_n))(x_i + f(x_1, \dots, 0, \dots, x_n))$$

Shannon's Reduction Theorems:
Theorem 3.12

(a) $x_i \cdot f(x_1, \dots, x_i, \dots, x_n) = x_i \cdot f(x_1, \dots, 1, \dots, x_n)$
(b) $x_i + f(x_1, \dots, x_i, \dots, x_n) = x_i + f(x_1, \dots, 0, \dots, x_n)$

Theorem 3.13

(a) $\overline{x}_i \cdot f(x_1, \dots, x_i, \dots, x_n) = \overline{x}_i \cdot f(x_1, \dots, 0, \dots, x_n)$
(b) $\overline{x}_i + f(x_1, \dots, x_i, \dots, x_n) = \overline{x}_i + f(x_1, \dots, 1, \dots, x_n)$

P2. Identity
There exist identity elements in B, denoted 0,1 relative to (+) and (·), respectively.
For all $x \in B, 0 + x = x + 0 = x, 1 \cdot x = x \cdot 1 = x$.

P4. Distributivity
Each operation (+), (·) is distributive over the other.
For all $x, y, z \in B$:
$$x + (y \cdot z) = (x + y) \cdot (x + z)$$
$$x \cdot (y + z) = (x \cdot y) + (x \cdot z)$$

P5. Complement
For every element $x \in B$ there exists an element $\overline{x} \in B$called the complement of x such that:
$$x + \overline{x} = 1$$
$$x \cdot \overline{x} = 0$$

Theorem 3.2: Null elements:
For each element x in a Boolean algebra:
$$x + 1 = 1$$
$$x \cdot 0 = 0.$$

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For the following problems, simplify the expressions using the postulates and theorems on the previous page.

1. Simplify: $\overline{w}\,\overline{x}yz + wxy + \overline{w}\,\overline{y} + x\overline{y} + \overline{x}\,\overline{y}$

$= x(wy + \overline{w}\,\overline{y} + \overline{y}) + \overline{x}(\overline{w}yz + \overline{w}\,\overline{y} + \overline{y})$ (Th. 3.11 (a))
$= x(\overline{y} + w) + \overline{x}(\overline{w}z + \overline{y})$ (Th. 3.13 (b))
$= x\overline{y} + xw + \overline{x}\,\overline{w}z + \overline{x}\,\overline{y}$ (P4)
$= \overline{y} + xw + \overline{x}\,\overline{w}z$ (P3, P4, P5, P2)

2. Simplify: $(w + x + \overline{y} + z)(w + x + \overline{y} + \overline{z})(w + \overline{x} + \overline{y} + z)(w + \overline{x} + \overline{y} + \overline{z}) \cdot (\overline{w} + \overline{x} + \overline{y} + z)(\overline{w} + \overline{x} + \overline{y} + \overline{z})$

$= z(w + x + \overline{y})(w + \overline{x} + \overline{y})(\overline{w} + \overline{x} + \overline{y}) + \overline{z}(w + x + \overline{y})(w + \overline{x} + \overline{y})(\overline{w} + \overline{x} + \overline{y})$ (Th 3.11(a))
$= (w + x + \overline{y})(w + \overline{x} + \overline{y})(\overline{w} + \overline{x} + \overline{y})$ (P4, P5, P2)
$= x(w + \overline{y})(\overline{w} + \overline{y}) + \overline{x}(w + \overline{y})$ (Th 3.11(a))
$= x\overline{y} + \overline{x}w + \overline{x}\,\overline{y}$ (P4, P5, P2, 3.11)
$= \overline{y} + \overline{x}w$ (P3, P4, P5, P2)

3. Simplify: $(x + z)(w + x)(\overline{y} + z)(w + \overline{y})$

$= x(\overline{y} + z)(w + \overline{y}) + \overline{x}zw$ (Th. 3.11(a))
$= x(yzw + \overline{y}) + \overline{x}zw$ (Th. 3.11(a))
$= x(zw + \overline{y}) + \overline{x}zw$ (Th. 3.13(b))
$= xzw + x\overline{y} + \overline{x}zw$ (P4)
$= zw + x\overline{y}$ (P3, P4, P5, P2)