

NAND Gate Realizations

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Symbolic Procedure

Using algebraic manipulations obtain a logic diagram consisting of only nand-gates for each of the following Boolean expressions:

$$1. f(w, x, y, z) = \bar{y} + w\bar{x} + \bar{w}x\bar{z}$$

$$\begin{aligned} &= \overline{\overline{\bar{y} + w\bar{x} + \bar{w}x\bar{z}}} \\ &= \overline{y \cdot w\bar{x} \cdot \bar{w}x\bar{z}} \\ &= \text{NAND}(y, \overline{w\bar{x}}, \overline{\bar{w}x\bar{z}}) \\ &= \text{NAND}(y, \text{NAND}(w, \bar{x}), \text{NAND}(\bar{w}, x, \bar{z})) \end{aligned}$$

$$1. f(w, x, y, z) = (w + y)(\bar{x} + \bar{z})(\bar{w} + \bar{x} + \bar{y})$$

$$\begin{aligned} &= \text{NAND}\left(1, \overline{(w + y)(\bar{x} + \bar{z})(\bar{w} + \bar{x} + \bar{y})}\right) \\ &= \text{NAND}\left(1, \text{NAND}((w + y), (\bar{x} + \bar{z}), (\bar{w} + \bar{x} + \bar{y}))\right) \\ &= \text{NAND}\left(1, \text{NAND}\left(\overline{w + y}, \overline{\bar{x} + \bar{z}}, \overline{\bar{w} + \bar{x} + \bar{y}}\right)\right) \\ &= \text{NAND}\left(1, \text{NAND}\left(\overline{w \cdot y}, \overline{x \cdot z}, \overline{w \cdot x \cdot y}\right)\right) \\ &= \text{NAND}\left(1, \text{NAND}(\text{NAND}(\bar{w}, \bar{y}), \text{NAND}(x, z), \text{NAND}(w, x, y))\right) \end{aligned}$$

Graphical Procedure

Convert the following Boolean expressions into logic diagrams and using the graphical procedures, convert the logic diagram into a logic diagram consisting of only nand-gates.

$$1. f(w, x, y, z) = \overline{w}(x\overline{y} + \overline{x}y) + z(x + y)$$

$$\begin{aligned} &= \overline{\overline{w}(x\overline{y} + \overline{x}y) + z(x + y)} \\ &= \overline{\overline{w}(x\overline{y} + \overline{x}y)} \cdot \overline{z(x + y)} = \text{NAND}(\overline{\overline{w}(x\overline{y} + \overline{x}y)}, \overline{z(x + y)}) \\ &= \text{NAND}(\text{NAND}(\overline{w}, (x\overline{y} + \overline{x}y)), \text{NAND}(z, x + y)) \\ &= \text{NAND}(\text{NAND}(\overline{w}, (\overline{\overline{x\overline{y} + \overline{x}y}})), \text{NAND}(z, \overline{\overline{x + y}})) \\ &= \text{NAND}(\text{NAND}(\overline{w}, (\overline{\overline{x\overline{y}} \cdot \overline{\overline{\overline{x}y}}})) , \text{NAND}(z, \overline{\overline{x \cdot \overline{y}}})) \\ &= \text{NAND} \left(\text{NAND} \left(\overline{w}, \text{NAND} \left(\overline{\overline{x\overline{y}}}, \overline{\overline{\overline{x}y}} \right) \right), \text{NAND}(z, \text{NAND}(\overline{x}, \overline{y})) \right) \\ &= \text{NAND} \left(\text{NAND}(\overline{w}, \text{NAND}(\text{NAND}(x, \overline{y}), \text{NAND}(\overline{x}, y))), \text{NAND}(z, \text{NAND}(\overline{x}, \overline{y})) \right) \end{aligned}$$

$$2. f(w, x, y, z) = (\overline{x} + y\overline{z})[w + (y + z)(\overline{y} + \overline{z})]$$

$$\begin{aligned} &= \text{NAND}(1, \overline{(\overline{x} + y\overline{z})[w + (y + z)(\overline{y} + \overline{z})]}) \\ &= \text{NAND} \left(1, \text{NAND}((\overline{x} + y\overline{z}), [w + (y + z)(\overline{y} + \overline{z})]) \right) \\ &= \text{NAND} \left(1, \text{NAND} \left(\overline{\overline{\overline{x} + y\overline{z}}}, \overline{\overline{\overline{w + (y + z)(\overline{y} + \overline{z})}}} \right) \right) \\ &= \text{NAND} \left(1, \text{NAND} \left(\overline{\overline{x \cdot \overline{y\overline{z}}}}, \overline{\overline{\overline{w \cdot y + z \cdot \overline{y} + \overline{z}}}} \right) \right) \\ &= \text{NAND} \left(1, \text{NAND} \left(\text{NAND}(x, \overline{y\overline{z}}), \text{NAND}(\overline{w}, \overline{\overline{y + z}}, \overline{\overline{y + \overline{z}}}) \right) \right) \\ &= \text{NAND} \left(1, \text{NAND} \left(\text{NAND}(x, \text{NAND}(y, \overline{z})), \text{NAND} \left(\overline{w}, \text{NAND} \left(1, \overline{\overline{y + z}} \right), \text{NAND} \left(1, \overline{\overline{y + \overline{z}}} \right) \right) \right) \right) \\ &= \text{NAND} \left(1, \text{NAND} \left(\text{NAND}(x, \text{NAND}(y, \overline{z})), \text{NAND}(\overline{w}, \text{NAND}(1, \overline{\overline{y \cdot \overline{z}}}), \text{NAND}(1, \overline{\overline{y \cdot z}})) \right) \right) \\ &= \text{NAND}(1, \text{NAND}(\text{NAND}(x, \text{NAND}(y, \overline{z})), \text{NAND}(\overline{w}, \text{NAND}(1, \text{NAND}(\overline{y}, \overline{z}), \text{NAND}(1, \text{NAND}(y, z)))))) \end{aligned}$$