m-Notation, M-Notation

Lecture 5 supplement

Boolean Formulas and Functions

- Example: $f(x, y, z) = (\overline{x} + y)z$
- Can be specified via a truth table.

X	Y	Z	f
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	1

Normal Forms

• Consider the function:

 $f(w, x, y, z) = \overline{x} + w\overline{y} + \overline{w} \,\overline{y}z$

- A literal is an occurrence of a complemented or uncomplemented variable in a formula.
- A product term is either a literal or a product (conjunction) of literals.
- Disjunctive normal form: A Boolean formula written as a single product term or as a sum (disjunction) of product terms.

Normal Forms

• Consider the function:

 $f(w, x, y, z) = z(x + \overline{y})(w + \overline{x} + \overline{y})$

- A sum term is either a literal or a sum (disjunction) of literals.
- Conjunctive normal form: A Boolean formula written as a single sum term or as a product (conjunction) of sum terms.

Canonical Formulas

• How to obtain a Boolean formula given a truth table?

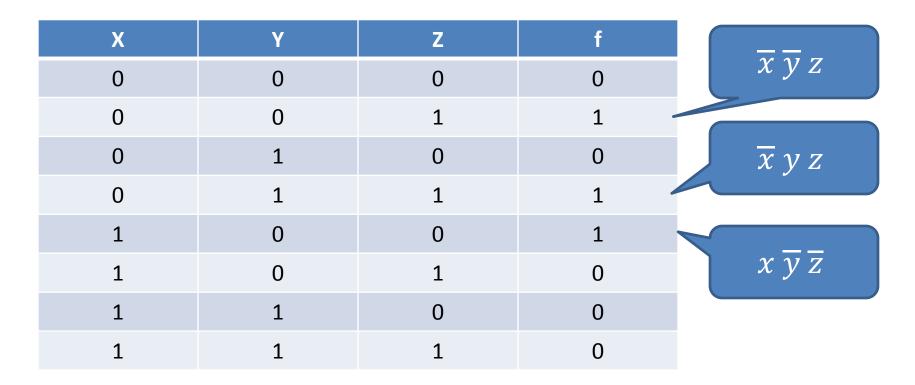
X	Y	Z	f
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	0

Minterm Canonical Formula

X	Y	Z	f	
0	0	0	0	$\overline{x} \overline{y} z$
0	0	1	1	
0	1	0	0	$\overline{x} y z$
0	1	1	1	
1	0	0	1	
1	0	1	0	$x \overline{y} \overline{z}$
1	1	0	0	
1	1	1	0	

 $f(x, y, z) = \overline{x} \, \overline{y} \, z + \overline{x} y \, z + x \, \overline{y} \, \overline{z}$

m-Notation



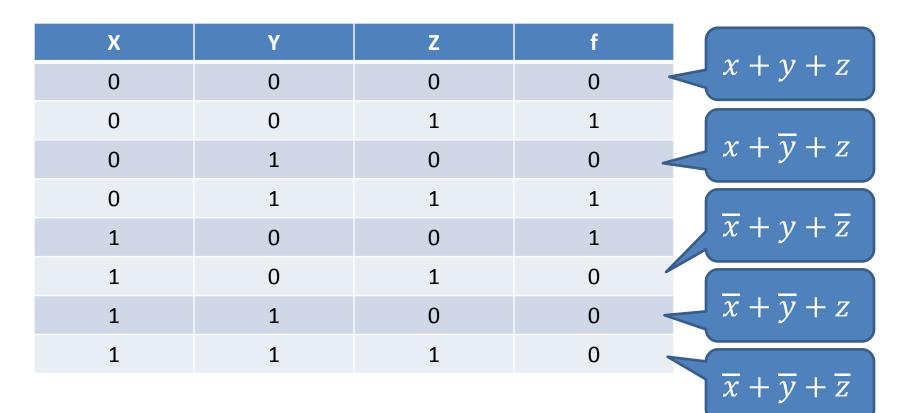
- f(x, y, z) can be written as $f(x, y, z) = m_1 + m_3 + m_4$
- $f(x, y, z) = \Sigma m(1, 3, 4)$

Maxterm Canonical Formula

X	Y	Z	f	
0	0	0	0	$\left\{ x+y+z \right\}$
0	0	1	1	
0	1	0	0	$\langle x + \overline{y} + z \rangle$
0	1	1	1	
1	0	0	1	$\overline{x} + y + \overline{z}$
1	0	1	0	
1	1	0	0	$\overline{x} + \overline{y} + z$
1	1	1	0	
				$\overline{v} \perp \overline{v} \perp \overline{z}$

 $f(x, y, z) = (x + y + z)(x + \overline{y} + z)$ $(\overline{x} + y + \overline{z})(\overline{x} + \overline{y} + z)(\overline{x} + \overline{y} + \overline{z})$

M-Notation



• f(x, y, z) can be written as $f(x, y, z) = M_0 M_2 M_5 M_6 M_7$

• $f(x, y, z) = \Pi M(0, 2, 5, 6, 7)$