Introduction to Cryptology ENEE459E/CMSC498R: Optional Homework 5

Due by beginning of class on 3/13/2018.

1. Recall our construction of CPA-secure encryption from PRF (Construction 3.30 in the textbook). Show that while providing secrecy, this encryption scheme *does not* provide message integrity. Specifically, show that an attacker who sees a ciphertext $c := \langle r, s \rangle$, but does not know the secret key k or the message m that is encrypted, can still create a ciphertext c' that encrypts $m \oplus 1^n$.

2. Say $\Pi = (\mathsf{Gen}, \mathsf{Mac}, \mathsf{Vrfy})$ is a secure MAC, and for $k \in \{0,1\}^n$, the tag-generation algorithm Mac_k always outputs tags of length $t(n)$. Prove that t must be super-logarithmic or, equivalently, that if $t(n) = O(\log n)$ then Π cannot be a secure MAC.
Hint: Consider the probability of randomly guessing a valid tag.

3. Consider the following MAC for messages of length $\ell(n) = 2n - 2$ using a pseudorandom function F: On input a message $m_0||m_1$ (with $|m_0| = |m_1| = n - 1$) and key $k \in \{0,1\}^n$, algorithm Mac outputs $t = F_k(0||m_0)||F_k(1||m_1)$. Algorithm Vrfy is defined in the natural way. Is $(\mathsf{Gen}, \mathsf{Mac}, \mathsf{Vrfy})$ secure? Prove your answer.

4. Let F be a pseudorandom function. Show that each of the following MACs is insecure, even if used to authenticated fixed-length messages. (In each case Gen outputs a uniform $k \in \{0,1\}^n$. Let $\langle i \rangle$ denote an $n/2$-bit encoding of the integer i.)
   (a) To authenticate a message $m = m_1, \ldots, m_\ell$, where $m_i \in \{0,1\}^n$, compute $t := F_k(m_1) \oplus \cdots \oplus F_k(m_\ell)$.
   (b) To authenticate a message $m = m_1, ..., m_\ell$, where $m_i \in \{0,1\}^{n/2}$, compute $t := F_k(\langle 1 \rangle || m_1) \oplus \cdots \oplus F_k(\langle \ell \rangle || m_\ell)$.